Thermionic Emission

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Summary

In this experiment, the emission of thermally excited electrons from tungsten will be studied. In addition, the work function of tungsten will be determined by taking into account the Schottky effect. The dependence of the resistivity of tungsten on its temperature will be observed. The results will be displayed graphically and the corresponding theoretical relations fitted.
During the experiment, the voltage V_A on the anode and the current I_C that heats the tungsten filament and cathode respectively can be varied, while the cathode voltage V_C and the anode current I_A, which is proportional to the amount of thermionically emitted electrons, are measured. The temperature T of the filament corresponding to different cathode currents will be determined with a pyrometer and the results will be compared to the theory of black body radiation.

Contents

1 Theory

1.1 Introduction

In order to study the thermionic emission of electrons from metals, we have to think of conductive electrons in a metal as a free electron-gas inside a periodic grid.

1.2 Energy distribution of electrons in the electron gas

The Maxwell-Boltzmann statistic is based on the assumption that each particle is from others particles distinguishable. Energy and momentum can take any possible value and the lowest possible energy level is reached for every particle when the temperature reaches the absolute zero of 0 K. In that case, the energy and the momentum are equal to 0, as in an ideal gas. However, various experiments (for example, the measurement of specific temperature of metals) have showed that a free electron gas has a zero-point energy: an energy greater than 0 even at temperatures close to 0 Kelvin.
Those scientific evidences lead to the introduction of a new statistic that had to be valid for electron gas: the Fermi-Dirac-statistic, which describes the statistical behaviour of fermions, such as electrons. The Fermi-Dirac-statistic is based on the Pauli exclusion principle, that says that every cell in the phase space can be occupied by not more than two particles, that must have different spin value. It follows, that even at a temperature $T_0 = 0$ K, every energy state is occupied up to the Fermi energy. The distribution function of the Fermi-Dirac statistic is

$$f(E) = \frac{1}{e^{\frac{E-E_F}{kT}} + 1} \tag{1}$$

Here, k is the Boltzmann constant and E_F is the Fermi energy. In Figure 1 the shape of the distribution is illustrated for different temperatures T. An accurate derivation of this equation can be found in [**Kittel**]

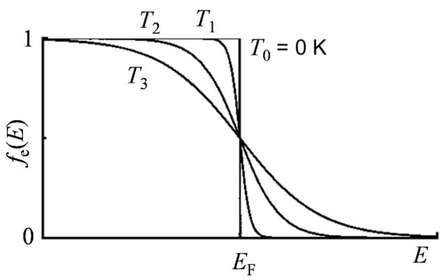


Figure 1: Fermi-Dirac distribution for different temperatures: $T_3 > T_2 > T_1 > T_0 = 0$ K. At the absolute zero temperature T_0, the probability that an electron has an energy below the Fermi energy E_F is equal to 1, while the probability that its energy is greater that E_F is zero. [**phdthesis**]

1.3 Binding energy of electrons in metal

In a first approximation, we represent the difference in potential between the inside of the metal and the vacuum as a finite square well, as portrayed in Figure 2. We assume, that every energy state, up to the Fermi energy E_F is occupied. The minimal energy W that is needed in order to emit an electron into the vacuum is called work function and corresponds to the energy difference between the vacuum E_0 and the Fermi energy E_F.

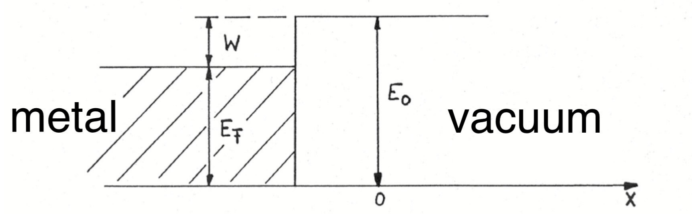


Figure 2: Finite potential barrier between the metal and vacuum at zero electric field.

The emitted current density j_s can be obtained from the flow of electrons through multiplication with the elementary charge $e = 1.6022 \times 10^{-19}$ C and through integration over the energy range

$$E_F + W \leq E_x \leq \infty \tag{2}$$

We then obtain

$$j_s = e \int_{E_F+W}^{\infty} \phi(E_x) \mathrm{d}E_x, \tag{3}$$

which results in the first version of the Richardson law

$$j_s = AT^2 \exp\left[-\frac{W}{kT}\right], \tag{4}$$

where A is a universal constant defined as

$$A = \frac{emk^2}{2\pi^2\hbar^3}. \tag{5}$$

For a more precise equation, instead of a finite square well, one has to consider the coulomb potential

$$V_1(x) = -\frac{e^2}{16\pi\varepsilon_0 x} \tag{6}$$

where ε_0 is the vacuum permittivity as well as the potential from an applied electric field F on the emitting surface

$$V_2(x) = -eFx \tag{7}$$

The second potential V_2 leads to the so called Schottky effect.
During the calculation of the emission current density, one should no longer integrate over $E_F + W \leq E \leq \infty$, but over $E_F + W - V_m \leq E \leq \infty$, where V_m is the maximum of the resulting potential V, which we define as

$$V(x) = V_1(x) + V_2(x) \tag{8}$$

The value of V_m can be found by differentiating and setting to 0.

$$0 = \frac{\mathrm{d}V(x)}{\mathrm{d}x} = \frac{\epsilon^2}{16\pi\varepsilon_0 x^2} - \epsilon F x_m = \frac{1}{4}\left(\frac{\epsilon}{\pi\varepsilon_0 F}\right)^{\frac{1}{2}}, \tag{9}$$

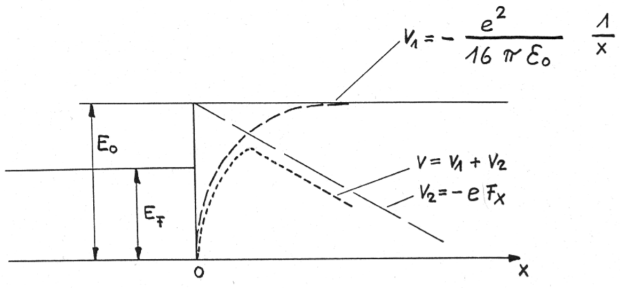


Figure 3: Potential $V(x)$ between a metal and the vacuum with a non vanishing electric field.

where $V(x_m) = V_m$. We can find out how big V_m is in this experiment

$$V_m = -\frac{e^2}{16\pi\varepsilon_0 x_m} - eFx_m = -\frac{1}{2}\left(\frac{e^3 F}{\pi\varepsilon_0}\right)^{\frac{1}{2}} \tag{10}$$

and by integrating (3) again we can find the emission current density j_s as

$$j_s = \frac{emk^2}{2\pi^2\hbar^3} \cdot T^2 \cdot \exp\left[-\frac{W - \frac{1}{2}\left(\frac{e^3F}{\pi\varepsilon_0}\right)^{\frac{1}{2}}}{kT}\right] \tag{11}$$

$$\Longleftrightarrow j_s = A \cdot T^2 \cdot \exp\left[-\frac{W - V_m}{kT}\right], \tag{12}$$

where we introduced A as in (5). One can see that the Schottky manifests itself in the Richardson equation as a reduction of the work function.

1.4 Final equation

From the measurements of the emission current density as function of the temperature, one can find the constant A and the work function W for the metal, using the final form of the Richardson equation (12). The value of A is often found to be different from the theoretical definition as in (5). This discrepancy can be explained by two considerations.

1. The surface of the metal is not perfectly flat and has a non homogeneous structure. These imperfections cause reflections of the electrons on the surface. This effect reduces the value of A to $D \cdot A$, where $D = 1 - \bar{r}$ and $\bar{r}$ is a reflection coefficient averaged over the energy. For metals we have $\bar{r} = 0.1$.

2. In Richardson's equation, the work function was assumed to be independent form the temperature. In reality, W shows in many metals a negative dependence on the temperature. By replacing W with $W_0 + \alpha T$ and by replacing A with $(1 - \bar{r}) \cdot A$, we can obtain a modified Richardson equation:

$$j_s = (1 - \bar{r}) \cdot A \cdot T^2 \cdot e^{-\frac{W_0 + \alpha T}{kT}} \tag{13}$$

This equation can be written in a simpler way, by introducing the constant $B = (1 - \bar{r}) \cdot A \cdot e^{-\frac{\alpha}{k}}$:

$$j_s = B \cdot T^2 \cdot e^{-\frac{W_0}{kT}} \tag{14}$$

The value of α has the magnitude of $5 \times 10^{-4}\,\mathrm{eV\,K^{-1}}$

1.5 Electronics

For this experiment, some basic equations from electronic are useful. First, Ohm's law:

$$R = \frac{V}{I} \tag{15}$$

and second, the equation for the resistivity ρ, which is a material dependent quantity

$$\rho = \frac{R \cdot A}{l}, \tag{16}$$

where A is the cross-section of the conductor and l its length. The unit of ρ is $\Omega{\cdot}\mathrm{m}$ [**resistivity-formula-fonte**].

2 Pyrometric Temperature Measurement

2.1 Introduction

High temperatures can be determined using thermal radiations laws that are valid for black-bodies. For non ideal black-bodies, a correction factor from the emissivity ε has to be taken into account. In the following, the most important emissions law are summarized. For more details take a look at [**Blackbody**].

2.2 Lambert's cosine law

Lambert found the following relation between the radiant flux $\dot{W} = \frac{\mathrm{d}W}{\mathrm{d}t}$ in Watt on a radiant source.

$$\mathrm{d}\dot{W}_\vartheta = \mathrm{const} \cdot \mathbf{F} \cdot \cos(\vartheta) \cdot \mathbf{F}' \cdot \frac{1}{\mathbf{R}^2} \tag{17}$$

Here, $\mathbf{F}$ is the surface area of the radiation source, $\mathbf{F}'$ is the surface area hit by radiation, $\mathbf{R}$ is the distance between $\mathbf{F}$ and $\mathbf{F}'$ and ϑ is the angle between $\mathbf{F}$ and $\mathbf{F}'$. The radiant intensity J_ϑ (in Watt) can be defined with the help of the solid angle $\mathrm{d}\Omega = \frac{\mathbf{F}'}{\mathbf{R}^2}$

$$J_\vartheta = \frac{\mathrm{d}\dot{W}}{\mathrm{d}\Omega} \tag{18}$$

By dividing the radiant intensity with the projection of $\mathbf{F}$, we can obtain the radiance S in Watt per squared meter.

$$S(\vartheta) = \frac{J_\vartheta}{\mathbf{F} \cdot \cos(\vartheta)} = \mathrm{const} \cdot \mathbf{F} \cdot \cos(\vartheta) \cdot \mathbf{F}' \cdot \frac{1}{\mathbf{R}^2} \cdot \frac{R^2}{\mathbf{F}'} \cdot \frac{1}{\mathbf{F} \cdot \cos(\vartheta)} = \mathrm{const}. \tag{19}$$

2.3 Planck's law

Planck found a relation between the spectral radiance of a black-body S, its temperature T and the wavelength λ of the emitted radiation.

$$S_\lambda(\lambda, T) = \frac{2hc^2}{\lambda^5 \Omega_0} \cdot \frac{1}{e^{\frac{hc}{\lambda k T}} - 1} \tag{20}$$

In equation (20) $\Omega_0 = 1$ sr, h is the Planck constant and c is the speed of light.

2.4 Wien's displacement law

Wien's displacement law can be obtained by differentiating equation (20) with respect to λ and setting the derivative equal to zero. Wien was able to find a relation between the peak of the wavelength $\lambda_{\max}$ and the temperature. Wien's displacement law states that the black-body radiation curve for different temperatures will peak at different wavelengths that are inversely proportional to the temperature. Wien's displacement law states that the peak of the wavelength times the temperature is constant:

$$\lambda_{\max} \cdot T = 2.88 \times 10^{-3} \text{ m}\cdot\text{K}. \tag{21}$$

2.5 Stefan–Boltzmann law

The Stefan–Boltzmann law can be obtained by integrating equation (20) over every wavelength. It gives the radiant flux of a black-body $\dot{W}$, which corresponds to the emitted power P, as a function of its temperature and its surface area F. For a black-body, the following equation holds:

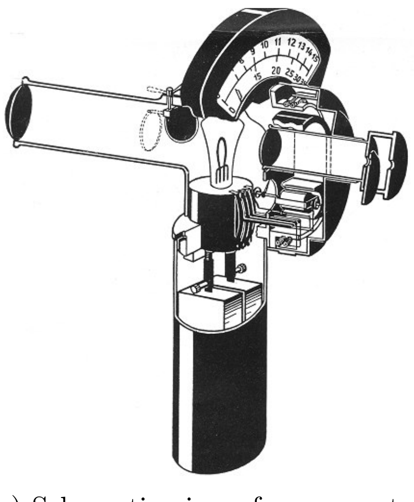
$$P = \dot{W} = \sigma \cdot F \cdot T^4 \tag{22}$$

Where $\sigma = \frac{2\pi^5 k^4}{15c^2h^3} = 5.670 \times 10^{-8}$ W/m^2·K^4 is the Stefan-Boltzmann constant. For a non ideal black body, the emissivity ε has to be added to the Stefan Boltzmann law

$$P = \sigma \cdot \varepsilon \cdot F \cdot T^4. \tag{23}$$

3 Pyrometry

The temperature of the incandescent cathode will be measured with a pyrometer. The radiant flux of the cathode will be compared to known radiant flux of a comparison source at a fixed wavelength of 0.65 µm. Students have access to two different types of pyrometers, depending on the work station. The first model is the Cambridge and the other model available is the Pyropto. The structure is identical. In both, one have



(a) Schematic view of a pyrometer.

(b) Picture of a pyrometer.

Figure 4: In both picture, it is shown a Pyropto pyrometer, the Cambridge one is almost identical. Adapted from [**pyrometer**].

to compare the two light sources they see. The image of the incandescent cathode gets projected with the help of a lens on the plane of a comparison filament. Through a magnifying glass it is possible to compare both filaments. The amperes flowing through the light bulb in the pyrometer can be adjusted. An amp meter shows the value of the temperature. This value needs to be corrected with the graph in Figure 6 Using a smoked glass the temperature range can be increased for higher temperatures. The wavelength that the red filter let pass is 0.65 µm.

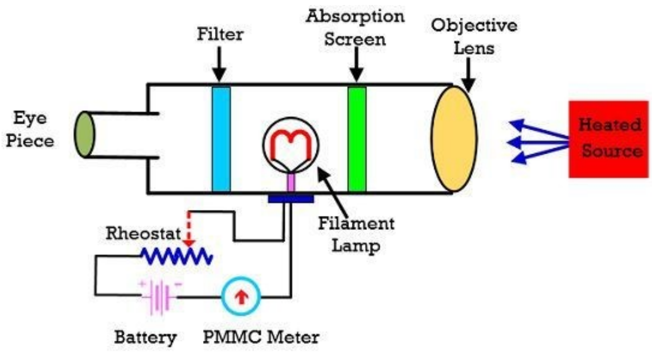


Figure 5: Circuit of a pyrometer. Here, the inside of a pyrometer is shown. Adapted from [**immaginecolorata**].

Even if they appear bright, looking directly in a pyrometer is not dangerous for your eyes. If the brightness becomes too high, a filter can be switched in, the brightness will fade and the color of the light will appear reddish.

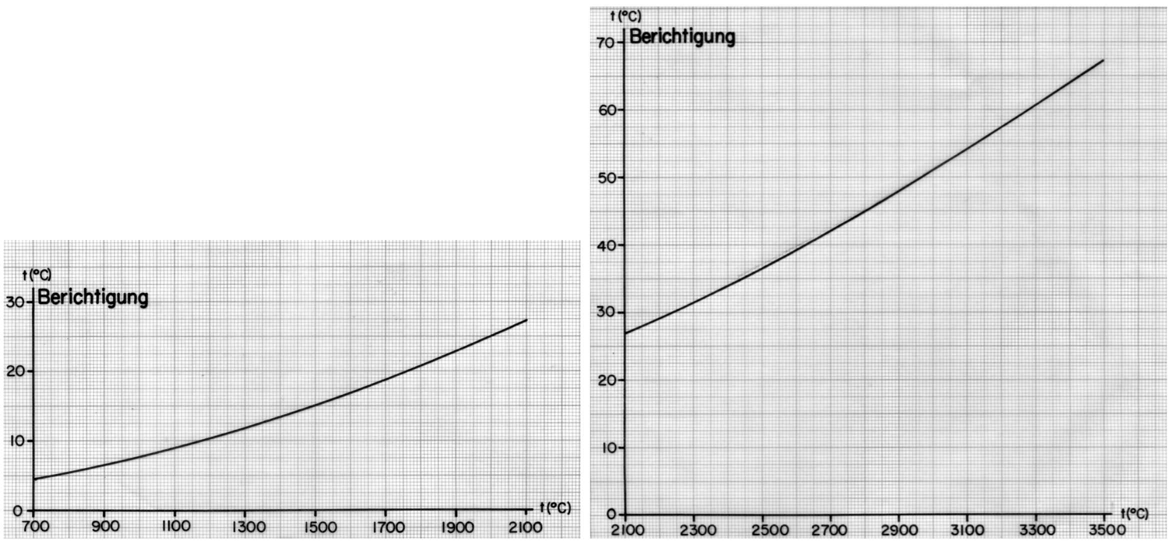


Figure 6: On the x-axis, the measured temperature, on the y-axis, the correction. For example, if a one measures 1500 °C, the corrected temperature is 1500 °C + 15 °C = 1515 °C.

3.1 Influence of the heat dissipation through the holders of the tungsten filaments (Optional)

By making a few assumptions, it is possible to calculate the temperature distribution [**Worthing**]. The filament has to be long, meaning that we can consider only one end of the filament. The filament will be approximated as a cylinder in vacuum. We also assume that the temperature over the entire cross-section of the filament is the same and that deviation are up to 0.25 degrees. With the resistivity ρ, the thermal conductivity k and the total emissivity ε we can find Joule's heat as

$$j^2 \rho \pi r^2 \mathrm{d}l \tag{24}$$

Thermal conductivity:

$$\pi r^2 \mathrm{d}l \cdot \left(k \cdot \frac{\mathrm{d}^2 T}{\mathrm{d}l^2} + \frac{\mathrm{d}k}{\mathrm{d}T} \cdot \left(\frac{\mathrm{d}T}{\mathrm{d}l} \right)^2 \right) \tag{25}$$

The radiation is

$$2\pi r \sigma \varepsilon T^4 \mathrm{d}l \tag{26}$$

In the end we obtain:

$$j^2 \rho \pi r^2 \mathrm{d}l + \pi r^2 \mathrm{d}l \cdot \left(k \cdot \frac{\mathrm{d}^2 T}{\mathrm{d}l^2} + \frac{\mathrm{d}k}{\mathrm{d}T} \cdot \left(\frac{\mathrm{d}T}{\mathrm{d}l} \right)^2 \right) = 2\pi r \sigma \varepsilon T^4 \mathrm{d}l \tag{27}$$

In equation 27, j is the current density, r is the radius of the filament and $\mathrm{d}l$ is the length (both are given in Figure 8). Interested students can solve equation (27).

4 Experiment

Follow the following steps to gather the data you need for the analysis. Cathode current I_C and voltage V_C as well as anode current I_A and voltage V_A are measured each with a multimeter.

- Switch on all the devices and build the circuit shown in Figure 7
- Choose a rather high cathode current (around 1.2 A) and find the isoelectric point (read next section for details).
- Read the temperature of the tungsten wire with the pyrometer for different cathode currents I_C between 0.9 A and 1.3 A in steps of 0.05 A. Here, one turns the screw (3) until the luminosity of the tungsten wire and the reference wire is the same and thus, one can read the temperature on the scale of the pyrometer. Because this method has a high imprecision, measure each temperature at least 6 times and then average it. Try starting both with a brighter reference wire and lower its brightness and with a lower brightness of the reference wire and then increase it. With two

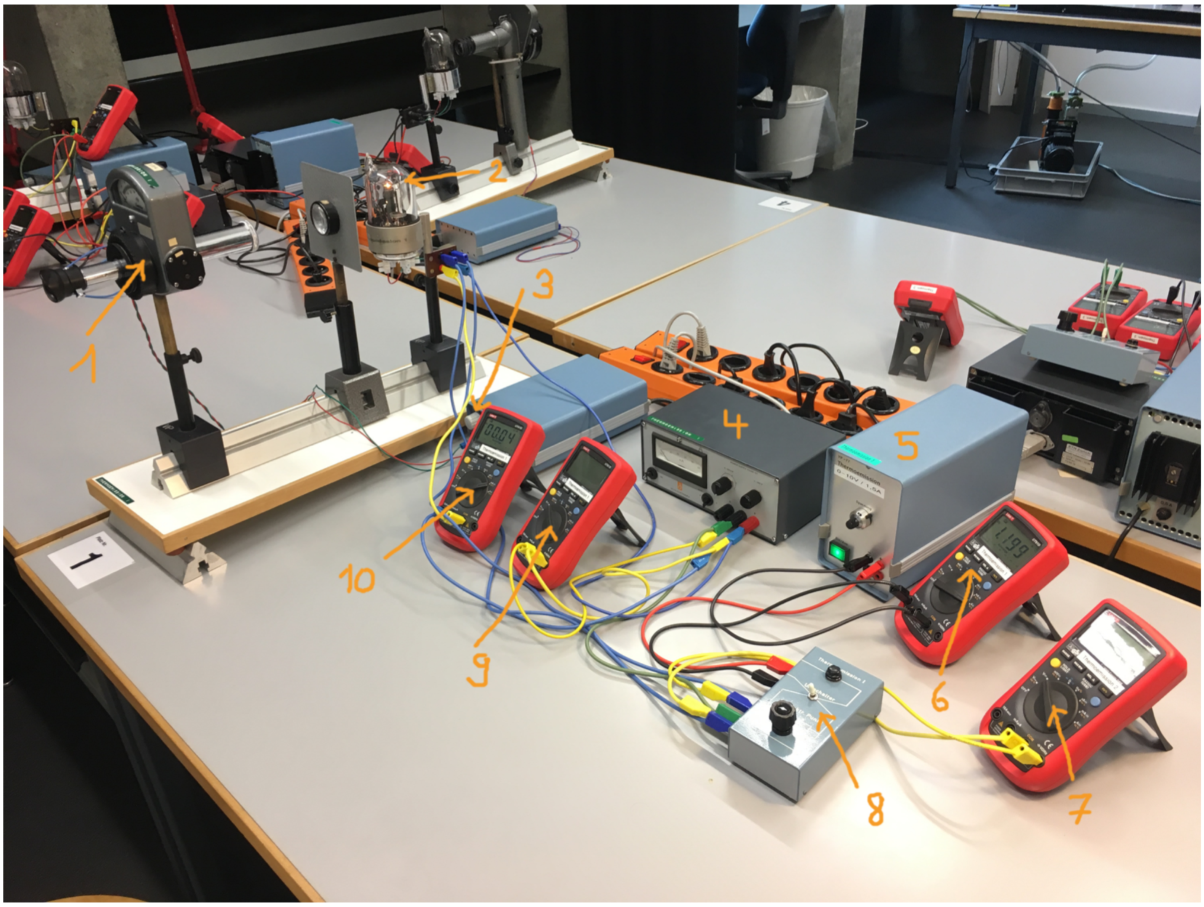


Figure 7: Setup of the experiment. The components are: pyrometer (1), tungsten wire (2), screw to adjust the luminosity of the reference wire in the pyrometer (3), power supplier which can vary the anode voltage (4), power supplier which generates the cathode current (5), multimeter for the cathode current (6), multimeter for the cathode voltage (7), changeover switch (8), multimeter for the anode voltage (9) and multimeter for the anode current (10).

different lenses, the used pyrometer can measure temperatures in the spectrum of around 1000 K up to more than 2500 K.

- Do a first measurement series for a fixed anode voltage V_A (e.g. just 0 V) by varying the cathode current I_C and thus also the temperature T. Use cathode currents Take down measurements of the anode current I_A as well as the cathode voltage V_C for each cathode current I_C between 0.9 A and 1.3 A in steps of 0.05 A.

- Repeat this measurement series for different anode voltages V_A between 0 V and 300 V in steps of around 30 volts. Now you only need to write down the anode current I_A, because the relation between I_C and V_C is independent of the anode voltage.

4.1 Isoelectric Point

The isoelectric point represents position of the ground. The ground should be in the middle of the filament so that the emission is equally distributed through the whole filament. To find it, the circuit in Figure 7 has to be build. Once this is done, the current through the cathode I_C must be kept high and the anode voltage V_A must be kept low. With a mechanical switch (shown as 8 in Fig. 7), the direction of the current can be inverted. The isolectric point is found, when the absolute value of the anode current I_A stays unchanged after changing the direction of the current with the switch. If the absolute value of I_A changes after changing the direction of the current, the knob (a potentiometer) should be turned and the current should be switched again until the isoelectric point is found.

5 Data Evaluation

It is suggested to use SI units for all analysis. Overall there are four different relations to examine.

- First, correct the measured temperature with the curve in Figure 6. For more precise results, you can optionally take also into account the reflection of light at the glass of the tube and at the lens inside the pyrometer. Then, plot the corrected temperature as a function of I_C.
  Compare the measured temperature of the tungsten filament with the values calculated with the Stefan-Boltzmann law (23) if you assume that all of the electrical power $P = V_C \cdot I_C$ gets radiated. Respect the emissivity ε of tungsten, as the filament is not a ideal black body.

- Plot the cathode voltage V_C in dependence of the cathode current I_C. Fit a linear function on the data. What physical interpretation has the slope of it? If possible fix the y-axis intersection of the fit to be 0.
  Now also plot the resistivity ρ as defined in (16) for tungsten in dependence of the temperature T and compare it to the literature.

- Plot $\log(\frac{j_s}{T^2})$ in dependence of $1/T$ and determine the work function W_0 of tungsten as well as the constant A in Richardson's equation defined as (5) and discuss the deviations between the theory and the obtained results. Determine W_0 and A by deriving them from the fit parameter of a linear function in the plot and Richardson's law (14).

- Discuss the impact of the Schottky effect by plotting $\log(I_A)$ over $\sqrt{U_A}$ and show the linear relation between the two as predicted in equation (12).

6 Dimensions of the experimental apparatus

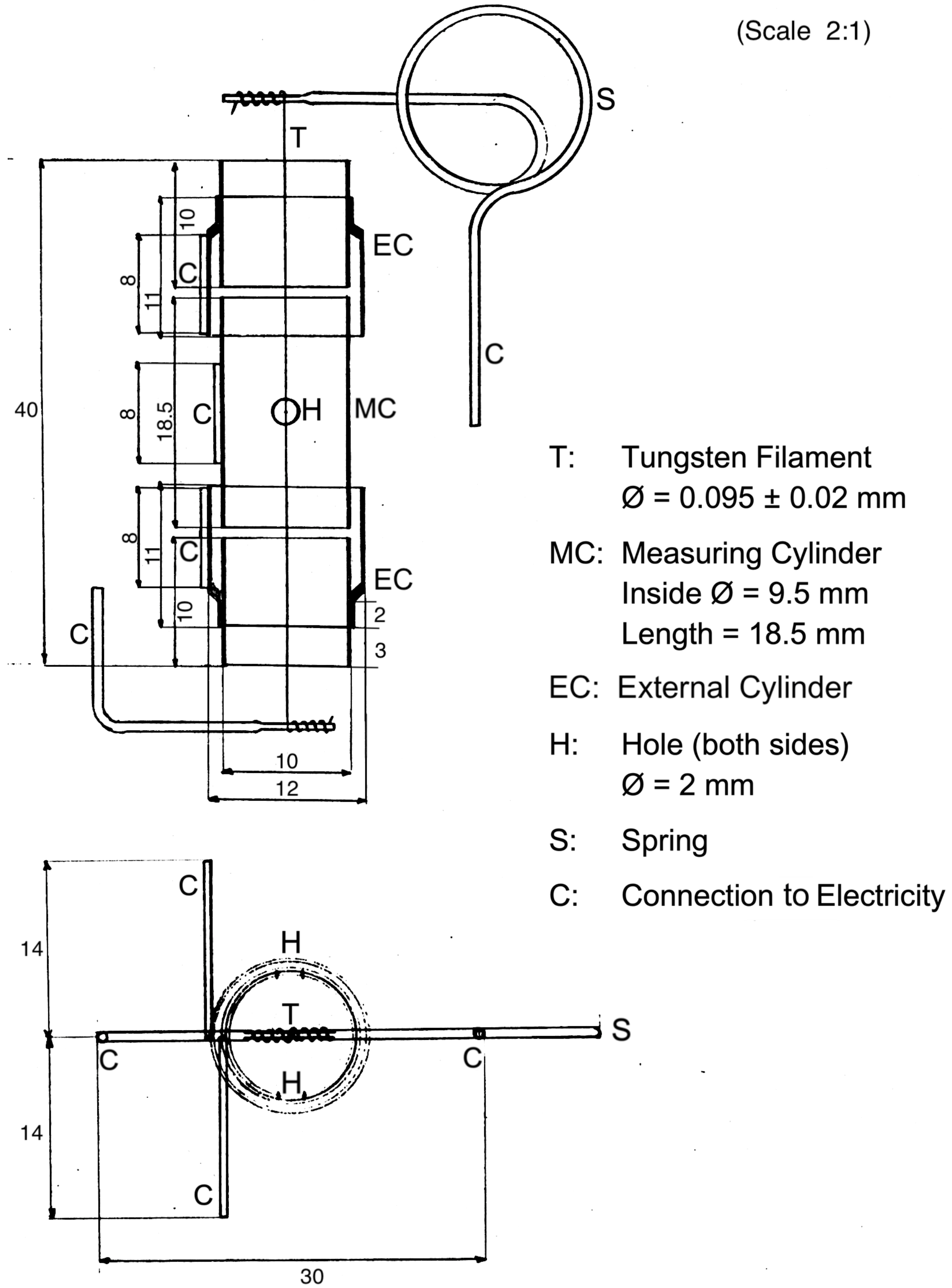


Figure 8: Useful dimensions of the experimental apparatus. The image on the top represents a side view of the experimental apparatus, the image below is a view of the same apparatus from above. Trough hole H passes the light.

Safety Instructions

As always, do not bring any liquid or food inside the lab. While building the circuit and while modifying it if needed, always turn off the power to the circuit. To prevent the filament from breaking, do not apply a current above 1.3 A to the filament ($I_C < 1.3$ A).

Further Literature

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