Laser - HeNe (Manual)

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1 Introduction

In this experiment a setup with a helium-neon laser and a spherical Fabry-Perot interferometer (SFP) will be used to investigate multi-mode and singlemode laser operation. The intensity patterns of the laser in both operation modes will be recorded for different cavity lengths using the SFP. The frequency separation between laser modes will be analysed as a function of the resonator length. Ultimately, the length and the free spectral range of the SFP will be determined.

2 Theoretical basics

2.1 Helium-neon laser

The term laser is the abbreviation of Light Amplification by Stimulated Emission of Radiation. A laser device consists of two main components: An amplifying (active) medium and an optical resonator. In this experiment, a gas laser containing a mixture of helium and neon is used.

Active medium The active medium - also called "gain medium" - consists of a material that allows electromagnetic waves passing through it to amplify due to the process of stimulated emission. In the case of the helium-neon laser, the active medium is a mixture of helium and neon gas (ratio 10:1) inside an electrical discharge. Due to inelastic collisions with energetic electrons, the Helium atoms are excited from the ground state to higher, metastable energy states, such as 2^3s and 2^1s . As can be seen in figure 1, these are almost at the same level as the 3s and 2s levels of neon. When an excited helium atom collides with a neon atom in the ground state, the excitation energy can be transferred from the He-atom to the Ne-atom. This leads to a selective excitation of higher levels in the Ne-atom (for more information see [laser]) which are also metastable. In this manner so-called "population inversion" is achieved. It occurs when a system is in a state where more atoms exist in higher excited states, than in lower excited states. By the process of stimulated emission, the Ne-atoms transfer back to lower energy states and the system becomes capable of amplifying light waves travelling through the active medium.



Figure 1: Grotrian diagram of helium and neon ([laser])

Optical resonator The gain medium is placed between two plane or spherical mirrors, which form the optical resonator. One of the two mirrors has a an almost total reflectivity ($\mathbf{r} \approx 100$ %), while the other one has a slightly lower reflectivity ($\mathbf{r} \approx 99$ %) and therefore allows a small transmission. It is important to note that the standing waves inside the resonator must fulfill a specific condition: Their half wavelengths must "fit" inside the resonator. So if d_{LR} denotes the resonator length and λ the wavelength, then $d = q \cdot \frac{\lambda}{2}$, where $q \in \mathbb{N}$. Using $c = \nu \cdot \lambda$ (where c is the speed of light and ν the frequency of the wave), we can see that only discrete frequencies of

$$\nu_q = q \cdot \frac{c}{2d_{LR}} \tag{1}$$

are allowed inside the resonator. The standing waves with these specific eigenfrequencies are called modes. Taking the difference between two successive frequencies, we find

$$\Delta \nu_{LR} = \nu_{q+1} - \nu_q = \frac{c}{2d_{LR}} \tag{2}$$

for the frequency separation of laser modes. This quantity is called the free spectral range (FSR) of the laser.



Figure 2: The standing waves in an optical resonator

Single-mode and multi-mode operation In the section above, the picture of waves inside a resonator was put in a simplified way. To be more precise, the cavity modes in a laser are given by the solution of the paraxial Hemlholtz equation ([**beams**]):

$$\frac{\partial^2 \psi}{\partial^2 x} + \frac{\partial^2 \psi}{\partial^2 y} = 2ik\frac{\partial \psi}{\partial z},\tag{3}$$

where $k = \frac{2\pi}{\lambda}$ is the wavenumber. In the single-mode operation, the laser operates on a single transversal resonator mode, which is given by the fundamental solution of (3), a Gaussian mode:

$$\psi(x, y, z) \propto exp\left(-\frac{r^2(x, y)}{w(z)}\right).$$
 (4)

In the multi-mode operation, the laser beam is composed of many different transversal modes (TEM). The shapes of different TEM modes are illustrated in figure 3, where the colors show where laser radiation corresponding to the different modes is detected. The lowest transversal mode corresponds to the Gaussian profile and has the smallest diameter compared to other modes.



Figure 3: Propagation modes for (a) rectangular and (b) cylindrical geometry. The top left profile shows a Gaussian mode in both cases. (source: **[poster**])

2.2 Fabry-Perot interferometer

A Fabry-Perot interferometer consists of two mirrors of high reflectivity, which form an optical resonator. It transmits only wavelengths that are in resonance with the cavity. In this experiment this property will be used to analyse the modes of the He-Ne laser. Depending on the shape of the mirrors, one can distinguish between different types of Fabry-Perot interferometers.

Plane-parallel Fabry-Perot interferometer (PFP) The resonator of the plane-parallel Fabry-Perot interferometer consits of two plane mirrors opposing each other. Figure (4) shows the path of an incoming wave in a PFP. We define the reflection coefficient $r := A_r/A_0$ and the transmission coefficient $t := A_t/A_0$, where A_0 denote the amplitude of the incoming wave and A_r and A_t the amplitudes of the reflected and transmitted wave respectively.

With α being the angle of incidence (the angle between the incoming wave and the normal of the PFP), the path difference between two successive waves is given by

$$\Delta s = \frac{2d}{\cos\alpha},\tag{5}$$

which yields the corresponding phase difference

$$\Delta \phi = \frac{2\pi}{\lambda} \cdot \frac{2d}{\cos\alpha}.$$
 (6)

Everytime a wave is reflected or transmitted, we have to account for the phase



Figure 4: The plane-parallel Fabry-Perot interferometer (PFP)

difference and the reflection, respectively the transmission coefficient. After n reflections, the transmitted wave has an amplitude of

$$u_n = t^2 \cdot r^{2n} \cdot e^{in\Delta\phi}.$$
(7)

Therefore, the amplitude of the superposition of all transmitted waves on the right side of the PFP is

$$u_{res} = t^2 (1 + r^2 e^{i\Delta\phi} + \dots + r^{2n} e^{in\Delta\phi} + \dots) = t^2 \cdot \frac{1}{1 - r^2 e^{i\Delta\phi}},$$
(8)

which was simplified using a geometric series. Because we are interested in the intensity of the light, which is proportional to $|u_{res}|^2$, we define new reflection and transmission coefficients for the intensities as $R := |r|^2 = I_r/I_0$ and $T := |t|^2 = I_t/I_0$, where I_0 is the intensity of the incident wave and I_r and I_t denote the intensities of the reflected, respectively transmitted wave. Because of energy conservation, it holds that R + T = 1.

Each Fabry-Perot interferometer is characterised by its finesse F. This indicates how many light waves interfere with each other, which in turn determines the sharpness of the peaks. It is defined by

$$F = \frac{\pi\sqrt{R}}{1-R}.$$
(9)

Using the expression for the finesse, the resulting transmission is

$$T_{res} = \frac{I_{res}}{I_0} = \frac{1}{1 + (2F/\pi)^2 \sin^2(\Delta\phi/2)}.$$
(10)

Transmission minima and maxima depend on the phase shift $\Delta \phi$: The transmission maxima are found at a phase shift of $\Delta \phi_{min} = 2\pi q$ and the transmission maxima at a phase shift of $\Delta \phi_{max} = 2\pi q + \pi$ for $q \in \mathbb{N}$. Using expression (4), we find that transmitted waves only have discrete wavelengths which fulfill

$$q \cdot \lambda_q = \frac{2d}{\cos\alpha} \cdot q. \tag{11}$$

The corresponding frequencies $(c = \lambda \nu)$ are given by

$$\nu_q = \frac{c}{\lambda_q} = q \cdot \frac{c \cdot \cos\alpha}{2d}.$$
(12)

Under the assumption of $\alpha = 0$, the frequency difference of two transmitted waves is

$$\Delta \nu_{PFP} = \frac{c}{2d}.$$
(13)

This quantity is called the free spectral range of the PFP.

Spherical Fabry-Perot interferometer (SFP) Instead of plane mirrors, the resonator of a spherical Fabry-Perot interferometer (SFP) is composed of two spherical mirrors. Figure 5 shows the geometric ray paths of an incoming beam in the spherical SFP and the way it changes after a certain number of reflections. We claim that the paraxial optic law sin $\alpha_i = \alpha_i$ is valid and assume that the distance d between the mirrors is equal to their curvature radius ρ .



Figure 5: The geometric paths of light in a spherical Fabry-Perot interferometer (SFP)

Then the phase difference between a type-I and a type-II wave is given by

$$\Delta \phi = \frac{2\pi}{\lambda} 4d. \tag{14}$$

The resulting wave is composed of type-I waves, which show (4-n)-reflections, and type-II waves which have (4n + 2)-reflections. Hence the amplitude of the transmitted wave is given by

$$u_{res} = \overbrace{t^2(1 + r^4e^{i\Delta\phi} + \dots + r^{4n}e^{in\Delta\phi} + \dots)}^{type-I} + \underbrace{t^2r^2e^{i\Delta\phi/2}(1 + r^4e^{i\Delta\phi} + \dots + r^{4n}e^{in\Delta\phi} + \dots)}_{type-II}$$

If the two incoming waves are in phase (even symmetry), the amplitude of the transmitted wave is

$$u_{res} = t^2 \frac{1}{1 - r^2 e^{i\Delta\phi}} (1 + r^2 e^{i\Delta\phi/2}).$$
 (15)

and the resulting transmission intensity is then given by

$$T_{res} \approx \frac{T^2}{(1-R^2)^2 + 4R^2 sin^2(\Delta\phi/2)} \left((1-R)^2 + 4R\cos^2\frac{\Delta\phi}{4} \right).$$
(16)

The transmission is maximal for $\Delta \phi = 4q\pi$, where $q \in \mathbb{N}$, so the transmitted waves have discrete frequencies of

$$v_q = q \frac{c}{2d},\tag{17}$$

which yields the free spectral range

$$\Delta \nu_{even} = \frac{c}{2d}.$$
(18)

If the two waves have a phase shift of π , the transmission is given by

$$T_{res} \approx \frac{T^2}{(1-R^2)^2 + 4R^2 \sin^2(\Delta\phi/2)} \left((1-R)^2 + 4R \sin^2\left(\frac{\Delta\phi}{2}\right) \right), \quad (19)$$

and we find the same spectral range of

$$\Delta \nu_{odd} = \frac{c}{2d}.$$
(20)

The intensity patterns for the symmetric and antisymmetric oscillations are shown in figures 6 and 7.

If the incoming beam consists of the same number of symmetric and antisymmetric eigenoscillations, the transmission is a combination of expression (14) and (17) and has a free spectral range of

$$\Delta \nu = \frac{c}{4d}.$$
(21)

In this experiment we will use the expression



Figure 6: Transmission for symmetric oscillations



Figure 7: Transmission for antisymmetric oscillations

$$\Delta \nu_{SFP} = \frac{c}{4d} \tag{22}$$

for the free spectral range of the spherical Fabry-Perot interferometer.

	$\Delta v = c/4a$	
		j j

Figure 8: Transmission for the superposition of figure 6 and figure 7 equally.

3 Experimental set-up

The experimental setup is shown in figure 9. It consists of the He-Ne laser, the mode aperture, the SFP, a detector and an oscilloscope. The beam of the He-Ne laser is directed into the SFP interferometer. The latter only transmits laser frequencies that match one of its mode frequencies which depend on its cavity length (see section about SFP). The transmitted light reaches the photodiode (detector), which is connected to an oscilloscope. The latter is used to record the laser spectrum.



Figure 9: The experimental setup

3.1 Laser

The laser has a wavelength of $\lambda = 632.2$ nm and a maximum power of some mW.

Alignement Before starting the experiment, the laser must be aligned i.e. the optical axis of the resonator and the optical axis of the discharge tube must be collinear. If the laser is misaligned, no laser beam is produced. To align the laser, the auxiliary laser can be used. It is placed on the optical rail and aligned parallel to it. The height must be adapted to the height of the mirrors of the laser. Then the mirrors can be adjusted relative to this laser axis by autocollimation: The light beam emitted from the laser and the light beam reflected from the mirrors, a

pinhole can be used to verify if the laser beam is parallel to the axis of the optical rail. During the alignment process, it is usually sufficient to only move the back mirror (the one on the right in figure 9) and it should be avoided to move any other parts of the setup.

Using the laser during the experiment In the experiment, the length of the laser resonator will have to be varied. Once the laser is aligned at the smallest cavity length possible, only the back mirror should be moved in small steps, making sure the red laser pointer stays visible. One must be very careful, as the laser is easily misaligned.

Multi-mode and single-mode operation The He-Ne laser will have to be used in both multi-mode and single-mode operation mode. Without an additional aperture, the laser is in multi-mode operation mode and the intensity distribution on the oscilloscope should look similar to figure 10. To achieve single-mode operation, an iris diaphragm is placed on the optical rail and then slowly closed, until the intensity pattern looks similar to the one shown in figure 11.

		mn	q	
			m n q +	1 n q+2
	·····(D ⁻¹	Ĭ	
	m n q-2			m n q+3
	9			m n a+4
(JU	Ulo

Figure 10: Spectrum in multi-mode operation ([poster])

Figure 11: Spectrum in single-mode operation ([**poster**])

3.2 The spherical Fabry-Perot interferometer (SFP)

A detailed sketch of the SFP is shown in figure 12. A piezoceramic tube mounted between the two mirrors controls the length of the resonator. It deforms if an electric field is applied (for more information on piezoelectric materials see **[piezo]**):

$$dl_i = l_i \sum_{j=x,y,z} d_{ij} E_j, \tag{23}$$

where dl_i and l_i are the expansion (or contraction) and length along the axis i, d_{ij} the piezoelectric constants and E_j the components of the electric field vector. Applying an alternating voltage to the tube therefore changes the length of the resonator and consequently (see section about SFP) also the transmission frequencies and the free spectral range.

Figure 12: The SFP. 1: mirrors of the resonator, 2: focusing lens, 3: adjustment of the SFP resonator, 4: piezoceramic distance spacer

3.3 Oscilloscope

Channel 2 of the oscilloscope is connected to the photodetector. It is the voltage generated by the detector, which is directly proportional to the intensity of the incident laser beam. Therefore this axis does not have to be calibrated. Channel 1 records the signal sent to the piezoceramic that determines the SFP length. It corresponds to the voltage applied to the piezo and has to be calibrated (see first task of the experiment). Furthermore, there is a trigger input that synchronizes both channels to a common time given by the function generation.

Operating the oscilloscope The oscilloscope used is the Tektronix

TBS1052b-EDU (manual [manual]). In order to record the laser spectrum, the images on the oscilloscope can be saved on a USB flashdrive. The image is easily saved by pressing the 'save' button on the oscilloscope (manual page 64). It is important to note, that the sensitivity range of the oscilloscope is only $\pm 25V$ and channel 1 typically goes beyond this range when measuring the spectrum of the laser. So one must be careful to exclude data that is out of range for the data analysis.

3.4 Electronics

The unit LASER supplies the laser tube with an ignition voltage of $\sim 6 \text{kV}$, an operating voltage of $\sim 2.2 \text{kV}$, an operating current of $\sim 10 \text{mA}$, a filament voltage of 14V and a filament current of 300mA. The ignition should not commence until 3 sec after switching on the heating. The unit ELEKTRONIK generates the voltage for controlling the piezoceramics and the X, Y - control of the oscilloscope.

4 Tasks/Experiments to be performed

4.1 Recording intensity patterns

The laser has to be aligned at different cavity lengths (from the smallest to the greatest possible distance) and the spectrum has to be recorded for each cavity length - both in multi-mode and single-mode operation.

4.2 Calibration of the x-axis of the oscilloscope

From the intensity patterns in the multi-mode operation, the voltage difference between two peaks ΔU_M has to be determined for each cavity length. From the theory of piezoelectric materials, the expansion of the tube is known to be proportional to the applied voltage, so there is a constant α , which allows us to convert the distance between laser modes on the oscilloscope into frequencies:

$$\Delta \nu_{LR} = \alpha \Delta U_M. \tag{24}$$

Determine alpha by inserting the the expression derived for the free spectral range of the laser (equation 2) and by using a linear fit of the voltage differences as a function of the cavity length.

4.3 Determine the FSR of the laser

Using the previously calculated value for α , convert the voltage data displayed by the oscilloscope into frequencies, i.e. the free spectral range for each cavity length.

4.4 Determine the FSR and the length of the SFP using the data from the single-mode operation

For the data in the single-mode operation, we know that the distance between the peaks is determined by the cavity of the SFP as follows:

$$\Delta \nu = \frac{c}{4d_{SFP}}.$$
(25)

So in this case

$$\alpha \Delta U_{SM} = \Delta \nu = \frac{c}{4d_{SFP}}.$$
(26)

Use the previously determined value for alpha and the intensity distributions from the single-mode operation to determine the cavity length of the SFP.

4.5 Stability

In theory, the stability criterion ([stable]) for a SFP with cavity length L and mirrors with radii of curvature R_1 and R_2 is

$$0 \le \left(1 - \frac{L}{R_1}\right) \left(1 - \frac{L}{R_2}\right) \le 1.$$
(27)

In our case, $R_1 = R_2 = 1$ m. Examine the stability limits of the laser cavity by observing to what length the laser cavity can be extended while still obtaining a stable laser beam and compare it to the equation above. Do the results correspond to your expectation from theory?