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Anleitung Nr:

# Josephson-Effect (new setup)

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# 1 Introduction

In 1962, Brian D. Josephson theoretically investigated the tunnel effect between two superconductors and showed that in the case of an extremely thin ($\approx 10\mathring{A}$) insulating layer a current can flow without any voltage drop between the superconductors. This current is a supercurrent of electron pairs (Cooper pairs) which varies periodically with the macroscopic phase difference $\delta$ accross the junction. This effect is commonly referred as the dc-Josephson effect.

Josephson claimed, that if a voltage $V$ is applied to such a junction, then the superconducting tunnel current has a time-dependent oscillatory component. This phenomenon is called the ac-Josephson effect. The frequency at which the current oscillates is related to the applied voltage via the relation

$$f = \frac{2e}{h} V. \tag{1}$$

In this expression the frequency and the potential are coupled by the fundamental constants $e$ (electron charge) and $h$ (Planck's constant). The quantity

$$\phi_0 = \frac{h}{2e} = 2.067833758(46) \times 10^{-15}\ \mathrm{Vs} \tag{2}$$

is called the magnetic flux quantum whereas its inverse

$$\frac{1}{\phi_0} = 4.83593 \times 10^{14} \frac{\mathrm{Hz}}{\mathrm{V}} \tag{3}$$

is the Josephson constant.

Both dc and ac Josephson effect were experimentally verified shortly after their theoretical prediction. The ac-effect was used, among other things, for the so far most exact determination of the $h/e$ ratio (W.H. Parker, D.N. Langenberg, A. Denenstein and B.N. Taylor, Phys. Rev. 177, 639 (1969)).

# 2 Theoretical principle

An introduction to superconductivity and the Josephson effect can be found in the Feynman lectures [1] or in Tinkham's book [2]. Here we summarize the most important results needed for the experiment following Ref [1].

In a superconductor electrons couple to pairs with total spin zero. These pairs, called Cooper-pairs, are bosons which all condense to the same energy level $\Delta/2$ below the Fermi energy of the normal state. This condensed state can be described by the macroscopic wave function

$$\Psi = \sqrt{\rho}\, e^{i\theta}. \tag{4}$$

The phase $\theta$ is so far undetermined and the squared amplitude $\rho$ is not interpreted as a probability density but directly as the local particle density of Cooper pairs.

Now let two identical superconductors be separated by a thin insulating layer such that there is a real valued coupling $K$ between the two wave functions. Then the behavior of the junction can be described by the Schrödinger-like equation

$$\begin{aligned} i\hbar\frac{\partial}{\partial t}\Psi_1 &= U_1\Psi_1 + K\Psi_2 \\ i\hbar\frac{\partial}{\partial t}\Psi_2 &= U_2\Psi_2 + K\Psi_1, \end{aligned} \tag{5}$$

where the potentials $U_1$ and $U_2$ may be interpreted as electrostatic potentials and $2eV = U_1 - U_2$ corresponds to the applied voltage between the two superconductors.

Inserting wave functions of the form (4) into the system of equations (5) and assuming $\rho_1 \approx \rho_2$ one obtains the current

$$j = \partial_t\rho_1 = -\partial_t\rho_2 = j_0 \sin\delta \tag{6}$$

where $j_0 = 2K\rho_0/\hbar$, $\rho_0 = \sqrt{\rho_1\rho_2}$, and $\delta = \theta_2 - \theta_1$ is the phase difference of the two wave functions. The system of equations (5) gives

$$\delta(t) = \delta_0 + \frac{2e}{\hbar}\int_0^t V(\tau)d\tau. \tag{7}$$

The dc and the ac Josephson effects are described by eqs. (6) and (7), respectively.

**Dc-effect, $V = 0$:**

$$j = j_0 \sin\delta; \tag{8}$$

$$\delta = const. \tag{9}$$

The junction supports a finite dc current without any voltage drop up to a maximum value $j_0$. Tuning the phase difference changes the supercurrent and vice-versa fixing the current sets the phase difference.

**Ac-effect, $V \neq 0$:**

$$\frac{d\delta}{dt} = \frac{2eV}{\hbar} := 2\pi f_0; \tag{10}$$

$$\delta(t) = 2\pi f_0 t + const.; \tag{11}$$

As a result of this equation, a finite dc voltage applied to the junction leads to an alternating Josephson current flowing accross the junction. This current has the frequency $f_0 = 2eV/h$ which typically ranges from the GHz to the THz range for voltages in the range of 1 $\mu$V to 1 mV.

The radiation emitted by a Josephson contact is weak and the coupling of the contact to the environment is poor. Hence the radiation was only observed directly in very sophisticated experiments. In this experiment, we will look at an indirect manifestaion of the ac Josephson effect when the junction is irradiated with a high-frequency (GHz) alternating electric field (frequency $f_1$). In this situation, one observes steps in the $V$–$I$-characteristics called Shapiro-steps. The dc-voltage at which these steps set-in is constant and is an integer multiple of the frequency of the external field $V = nhf_1/2e$.

Indeed, the alternating field with frequency $f_1$ causes a potential difference $V_1(t)$ at the contact. Furthermore it boosts the quasi-particles tunneling, which is experimentally shown by the fact that $j_{max}$ is smaller than $j_0$ at the irradiated contact. Consequently there is also a dc-voltage $V_0$ at the contact. The Josephson equation reads

$$j = j_0 \sin(2\pi f_0 t + \frac{2eV_1}{hf_1} \cdot \sin(2\pi f_1 t)) \tag{12}$$

where $f_0 = 2eV_0/h$. This formula can be expanded into a Fourier series using the relations

$$\begin{aligned} \sin(z\sin(\theta)) &= 2\sum_{k=0}^{\infty} J_{2k+1}(z)\sin[(2k+1)\theta] & (13) \\ \cos(z\sin(\theta)) &= J_0(z) + 2\sum_{k=1}^{\infty} J_{2k}(z)\cos(2k\theta), & (14) \end{aligned}$$

where $J_k$ is the Bessel-function of $k-$th order. This expansion shows that time-independent (dc) terms exist, if

$$f_0 = nf_1, \quad \text{with } n \in \mathbb{N}^+ (n = 1, 2, 3, 4, ...). \tag{15}$$

*Intermediate task:* this calculation is recommended as an exercise.

**Intermediate steps.** Steps with non-integer ratios $f_1/f_0$ are also observed in this experiment. This is due to the fact that the Josephson equations (8) and (10) are idealized. The current at a realistic contact is not a pure sine-function of the phase $\delta$. It is only periodic in $\delta$ and can therefore be expanded into the Fourier series

$$j = \sum_{m=1}^{\infty} j_m \sin(m\delta). \tag{16}$$

If the steps are computed with this generalized Josephson equation, additional DC-current contributions occur in the cases

$$mf_0 = nf_1, \qquad \text{with } m, n \in \mathbb{N}^+. \tag{17}$$

# 3 The experiment

In this experiment, you will successively observe first the dc- and then the ac-Josephson effects. To this end, you will fabricate a superconducting tunnel junction by electrochemically oxidizing a Nb disk and mechanically sharpening a Nb tip. High sensitivity and low temperature measurements will allow you measuring $V(I)$ traces under microwave excitation tunable in frequency and power. **As a final goal you will extract a fundamental constant: the magnetic flux quantum $h/e$, evaluate its precision and accuracy**. The multiple steps necessary to achieve this goal are described below.

## 3.1 Josephson junction fabrication

Two steps are necessary to fabricate the Josephson junction used in this experiment. The first step consists in fabricating a sharp Nb tip, the second in oxidizing a Nb disk.

**Sharp Nb tip:** A sharp tip of Nb can be obtained by cutting a piece of Nb wire provided for the experiment with a tweezer. Care must be taken to obtain a **tip-like shape**. When the tip is made, do neither touch with fingers nor bend.

**Nb disk oxidation:** The oxidation of the Nb disk is achieved by electrochemistry while applying a constant voltage of 30 $V$ for 45 s accross the cathode made out of Cu and the anode made out of the Nb disk deeped into a $K_2SO_4$ (33%) solution. Polish the disk on both sides roughly with sandpaper and finely with the polishing powder mixed on a glass-plate with water. Rinse the disk extensively with water and let it dry on a tissue. Do not touch with bare hands anymore. While the disk is drying, prepare the Cu cathode by polishing it with sandpaper. Rinse the cathode extensively with water.

*Intermediate task:* Write down the equation for the redox.

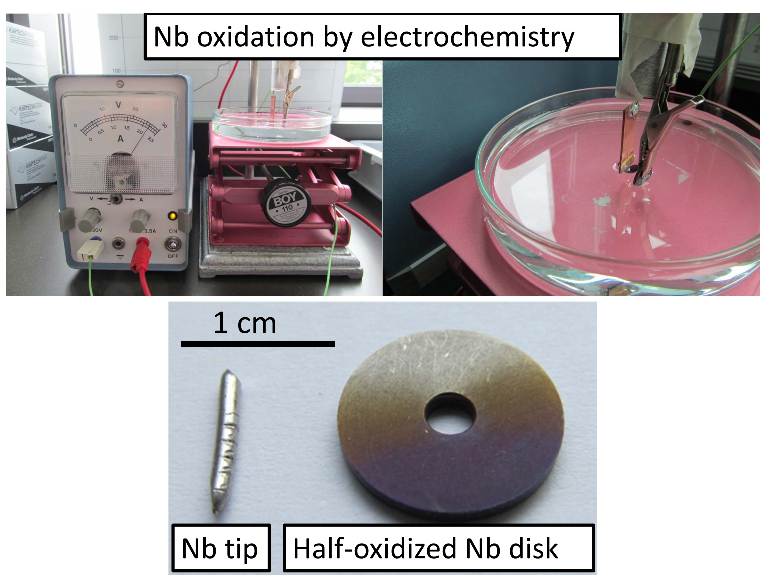


Figure 1: Josephson junction fabrication. Nb oxidation by electrochemistry and Nb tip sharpenning.

### 3.2 Mounting the Josephson junction on the insert

Once the two parts of the junction are clean and dry, mount them carefully in the sample area containing the differential screw mechanism, the microwave antenna and the 4 wires necessary for the 4-point measurements (see figure 2). Consult the assistant for details if necessary.

**Screwing cables + tip + disk:** First mount the disk by screwing it slightly with the plastic screw. Manipulate then the two wires coming from the bottom (see figure 2) in order to pinch them in between the washer and the conducting part of the disk. Tighten the screw to ensure good contact. The tip can be mounted as shown in figure 2 on the cantilever. Care must be taken to **neither damage the tip nor the disk** during the operation. Finally adjust the height of the cantilever for the tip to be close to but not touching the disk (few turns of the wheel on top of the stick should be enough, one full turn of he wheel changes the gap by 26.2 microns).

**Microwave antenna position:** Place the microwave antenna **as close as possible** to the tip ($\leq$ 0.5 mm without touching it) keeping at least 2 mm space between the antenna and the disk.

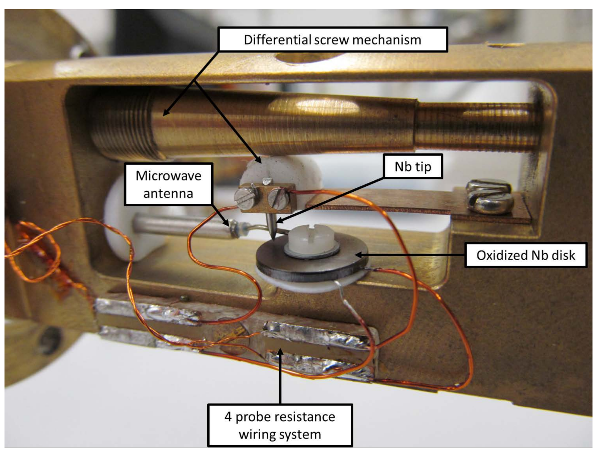


Figure 2: Picture of the sample space when all necessary elements are assembled.

### 3.3 Low temperature

Figure 3 shows how the insert should be handled for safe cooling and reliable measurements.

**O-ring sealing:** Screw tightly the sample shield on the measurement stick using an O-ring for sealing.

**He exchange gas:** Pump air out of the sample space using the vacuum pump for at least a few minutes. Flash with He gas, pump again and finally put a very small amount of He gas inside the sample space (exchange gas). Typically half of the balloon nozzle is enough ($\approx 1$ cm$^3$).

**Cooling procedure:** Put the measurement stick slowly into the liquid He dewar. During the whole procedure, wear **protecting gloves, glasses** and make sure that the **differential screw mechanism does not block** by slightly turning the corresponding wheel in both directions repeatedly, during cooldown.

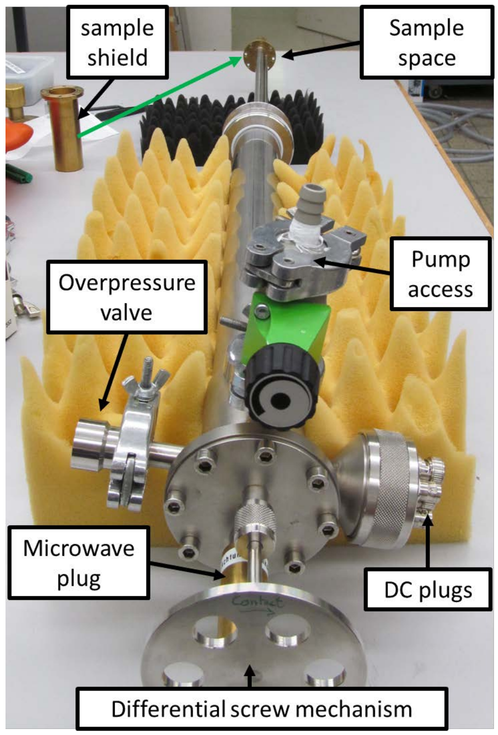


Figure 3: Global picture of the measurement insert.

## 3.4 Low noise measurement pocedure

**Microwave cable:** Plug the microwave cable on the SMA connector on the measurement stick (use corresponding torque wrench) **before** making any contact between the tip and the disk. Avoid **parasitic vibrations** of the dewar and cable during measurements.

**Making the Josephson contact:** The only reliable way of checking whether a Josephson contact is formed while turning the wheel (differential screw mechanism) is to **record the resistance of the junction while turning**. For this purpose, connect the setup as proposed in the next paragraph and approach the tip slowly towards the disk surface. As soon as the resistance changes, do not touch anything anymore and test the junction. If the resistance is still too high, push the tip a bit more against the disk.

**N.B.:** Be cautious while handling the setup not to damage the oxide layer by pushing too hard or not to shake the dewar.

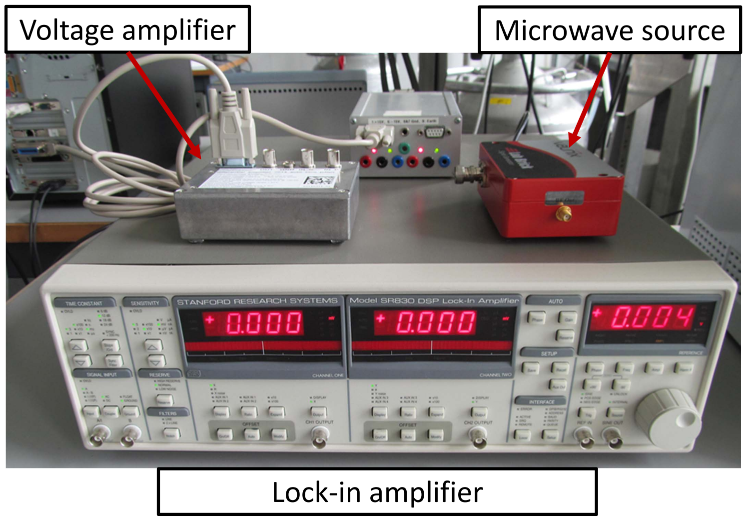


Figure 4: Measurement setup including a lock-in amplifier, a voltage amplifier and a microwave source.

**Measurement of the dc Josephson effect:** The $V(I)$ characteristic of the Josephson junction has to be acquired using the auxiliary output *AuxOut* of the lock-in amplifier as the current source (note that the auxiliary output is a voltage source) and the auxiliary input in combination with a low pass filter (small resistance RC filter) as the voltmeter. A voltage amplification stage with gain $G = 10^4$ is available. A GPIB to USB adapter stage links the computer to the lock-in amplifier, which is fully computer-controlled via Labview and the program *StepandLog*.

**Frequency optimization at maximum power using the dc Josephson effect:** The coupling of the microwave antenna to the Josephson junction is frequency-dependent. In order to maximimize this coupling, a preliminary experiment is performed. Apply a current $j = \pm 0.6 j_0$ to the junction with $j_0$ its critical current and record the voltage drop accross the junction as a function of the microwave frequency applied from 5 GHz to 10 GHz (computer-controlled microwave generator *Vaunix LMS 103*). The power must be maximum during this measurement (maximum power $P_{\text{max}} = +10$ dBm). The recorded curve will look like a succession of more or less pronounced peaks corresponding to the microwave modes coupling best to the junction. Identify the corresponding frequencies and use them for further measurements of the ac Josephson effect.

**Dc measurement of the ac Josephson effect for different power:** Using the previously determined frequencies that couple best to the junction, acquire $V(I)$ traces under microwave irradiation for a broad range of frequencies and power. A power dependence of the ac Josephson effect should reveal the Bessel-function behaviour of the Shapiro steps and the use of different microwave frequencies will enable verifying the step-height evolution with respect to the microwave frequency applied $V = nhf_1/2e$ as presented in the theoretical part of this manual.

**Ac measurement of the dc Josephson effect:** An interesting way of measuring the Josephson effect is to measure it with a lock-in amplifier. In such configuration, the measurement will correspond to the differential resistance $dV/dI$ of the junction. This may allow observing small features in the junctions' characteristics.

**Optional measurement: Ac measurement of the ac Josephson effect for different power:** The Bessel function behaviour of the voltage plateaus length with respect to the microwave power can be directly measured while acquiring a two-dimensional map of the differential resistance $dV/dI$ as a function of the dc current and the microwave power. Measure this map and qualitatively compare it to the Bessel function expression you may extract from equation 12.

## References

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*in the VP library