

Eidgenössische Technische Hochschule Zürich Swiss Federal Institute of Technology Zurich Advanced Physics Laboratory (VP) https://vp.phys.ethz.ch

Franck-Hertz Experiment Instructions

C. Dedes & P. Cerulo

April 2023, rev. David G. Schauer & Matteo Simoni

Contents

1	Introduction		
	1.1	Historical background	3
	1.2	Theoretical background	4
2	Pre	paratory questions	8
3	Experimental		9
	3.1	Procedure and setup	9
	3.2	Technical description of the setup	11
	3.3	Heating of the oven	12
	3.4	Retrieving the data with LabView	12
4	Measurements & analysis		
	4.1	Measurements in the laboratory	14
	4.2	Analysis of your recorded data (at home)	15
5	Rep	ort writing and structuring	16

1 Introduction

1.1 Historical background

The *Frank-Hertz* experiment was first carried out and published in 1914 by James Frank and Gustav Hertz. Despite the simplicity of the experimental set-up and the ease with which it could be performed, this experiment was the first to practically demonstrate the existence of discrete energy levels in atoms and molecules and show a conclusive proof of the atomic model, which Niels Bohr had proposed one year before. In the experiment, fast- accelerated electrons interact with the atoms of a gas through inelastic collisions and the excitation energy of the gas is studied as a function of the kinetic energy of the incident electrons. These findings earned the two authors the Nobel Prize in Physics in 1925 [1].



Figure 1: left: Comparison of the classical model of the atom and the proposed model by Niels Bohr (1913) in which the energy states an electron is able to occupy are quantized within the atom, proven by the *Franck- Hertz* experiment.

right: Portraits of James Franck and Gustav Ludwig Hertz [2].

In this experiment, mercury (Hg) is chosen as target gas, since it has a low electronegativity, reducing the probability of forming negative Hg ions when colliding with the incident electrons [3]. Moreover, mercury is easily available in high purity and also has a strong temperature dependence on the vapor pressure.

1.2 Theoretical background

The attempt to describe emission and absorption of light for a single atom with the laws of classical electrodynamics leads to essential difficulties. In classical physics, electrons can orbit around the atomic nucleus with arbitrary radii, leading to the possibility of having a continuous energy spectrum for the electrons in the field of the nucleus. However, the observed spectra of the atomic and molecular gases show well-defined lines at particular wavelengths that can only be explained by the fact that only discreet energies are allowed for the electrons. Moreover, since the electrons are accelerated by the electrostatic field of the nucleus, they should, according to classical physics, continuously emit radiation with a frequency ν corresponding to their orbital frequency ω (figure 1), lose energy (equation 1) and eventually collapse on the nucleus over timescales of less than 1 ns [4].

$$\nu = \frac{\omega}{2\pi} \tag{1}$$

In 1913, Niels Bohr made a first attempt to circumvent the conceptual obstacles posed by a classical interpretation of the atom. He therefore introduced a first concept of quantization in the form of two main postulates (shown in figure 1:

- 1. The electrons obey the classical equations of motion, although only well-defined and discrete orbits with energies E_n shall be allowed. These correspond to the energy levels of the atom.
- 2. When they are on their orbits (stationary states), the electrons don't emit any radiation (i. e. there's no loss of energy). However an electron can pass from an orbit with lower (negative) binding energy E_n (larger radius) into an orbit of larger (negative) binding energy E_m (smaller radius) by the emission of a quantum of radiation (photon),

$$\Delta E = E_n - E_m = h\nu, \qquad (2)$$

where h stands for the *Plank* constant (6.62607015 * 10^{-34} J s) and ν for the frequency of the electron. The reverse process is induced by the absorption of light.

The atomic model of Bohr can correctly (but not completely) explain the physics of the hydrogen atom, which contains only one proton and one electron. For poly-electronic atoms, such as mercury used in this experiment, the interactions between the orbiting electrons have to be taken into account. Therefore, a fully reliable explanation of the physics of these atoms can only be possible by writing and solving the corresponding *Schrödinger* equation. The atomic model of Bohr still constitutes a good approximation for these atoms and Bohr's two postulates can be reinterpreted in the context of the solution of the *Schrödinger* equation. From the point of view of modern quantum mechanics, the energies E_n correspond to the eigenvalues of the Hamiltonian, as expressed by the time-independent *Schrödinger* equation, where Ψ stands for the wave function of the electrons, $\hbar = \frac{h}{2\pi}$ for the reduced *Planck* constant (1.054571817... * 10⁻³⁴ J s), m for the reduced mass and V(x) for the potential energy operator

$$\frac{\hbar^2}{2m}\Delta\Psi(\mathbf{x}) + \mathbf{V}(\mathbf{x})\Psi(\mathbf{x}) = \mathbf{E}\Psi(\mathbf{x}). \tag{3}$$

In the case of the hydrogen atom, the energy eigenvalues of the single electron are given by

$$E_{n} = \frac{e^{2}}{2a_{0}} \frac{1}{n^{2}},$$
(4)

where e stands for the elementary charge $(1.602176634 * 10^{-19} \text{ C})$, n as a positive integer for the number of possible energy states and a_0 for the Bohr radius $(a_0 = \frac{h^2}{me^2} = 0.529 * 10^{-9} \text{ m})$.

In case of more electrons (e.g. Hg), the interaction between the electrons has to be taken into account and therefore the potential V(x) in equation 3 changes. The state with the lowest energy is called ground state and is stable, whereas states with higher energies are excited states. The transition

to a higher state requires the absorption of a discrete energy $\Delta E \sim \nu$. Such a transition can be achieved in different ways:

- Absorption of electromagnetic waves
- Collisions (either elastic or inelastic) between a free particle and the atom.
- Collisions of molecules due to thermal movement

Due to the coupling between the atom and the quantized electromagnetic field, the excited states are often unstable, and consequently, by spontaneous emission of a photon, the electron transitions back to the ground state. The state of an atomic electron is generally characterized by its values of energy, orbital angular momentum, spin and total angular momentum. The transitions are explained through a series of selection rules, establishing whether a certain state transition is allowed or not. These generally correspond to the laws of conservation of the four quantities listed above. The practical way to find whether a selection rule is observed in a state transition or not, is by calculating the elements of the transition matrix (see Ref. [5] for more details).

In Figure 2 a scheme of all the relevant states of the Hg atom is presented. From the ground state $6s^2 {}^{1}S_0$ ($6^{1}S_0$) different excited states can be populated using the energy of a photon with a specific wavelength. In the case of the excited state $6^{1}P_1$ (singulet state) the electron needs a transition energy equivalent to 184.9 nm. In order to populate one of the triplet excited states $6^{3}P_0$, $6^{3}P_1$ or $6^{3}P_2$ less energy is needed for the transition. The excitation from the ground state to $6^{3}P_0$ (265.6 nm = 4.67 eV) and to $6^{3}P_1$ (253.7 nm = 4.89 eV) is subject of this experiment.



Figure 2: Scheme of the ground state $6s^2 {}^1S_0$ and the excited states (singulet, triplet) of mercury taken from Ref. [6].

Note: Before starting the experiment, a preliminary discussion will take place between the assistant and the student. This will be used to assess the preparation of the latter, in order to establish if the student is sufficiently prepared to undertake the experiment. Therefore, answers to the theoretical questions in section 2 have to be prepared in advance. The information above and the papers given in the bibliography can be used to prepare for these questions and the experiment.

2 Preparatory questions

- 1. How can the distance between the maxima/minima (extrema) be explained and at which energies do they occur? Does this energy correlate to a specific energy in the possible transitions of mercury (Hg)?
- 2. Why will there be multiple maxima and minima?
- 3. Perform a simple calculation which shows the amount of energy in eV associated with a 253.7 nm and a 256.6 nm photon. Why are these wavelengths relevant in the energy-level diagram of Hg? What transitions do they describe?
- 4. (Optional, for better understanding of the Rapior Model, needed for your report later) Have a look at the paper of Rapior et al. [7] and try to understand their results about the spacing between the extrema in the *Franck-Hertz* curves. Is this spacing equidistant or does it increase/decrease? What is the relationship between the mean free path λ and the distance between the grids g_1 and g_2 (L) in correlation to that spacing?

3 Experimental

3.1 Procedure and setup

The experimental setup is schematically illustrated in figure 3.



Figure 3: Scheme of the *Franck-Hertz* experiment.

An adjustable oven is heated to vaporize the Hg, which is sealed airtight between the grids g_1 and g_2 . Electrons are emitted at a hot cathode, preaccelerated with a small voltage U_1 applied to grid g1 that removes the space charges around the cathode, and accelerated by a linearly increasing voltage U_2 between the grids g_1 and g_2 . After passing the mesh grid g_2 , they reach the anode A where they are detected. A small braking voltage U_3 is applied between the mesh grid g_2 and the anode A. While they are being accelerated between g_1 and g_2 , the electrons can undergo collisions with Hg atoms. We distinguish between elastic collisions and inelastic collisions. During an elastic collision between an electron and an Hg atom, the total kinetic energy is conserved and just gets redistributed between the two particles. During an inelastic collision, the total kinetic energy is not conserved. Electrons that have lost their kinetic energy due to inelastic collisions and have not been sufficiently accelerated after the last collision, are blocked by the voltage U_3 and are therefore not able to reach the anode. For a fixed pre-acceleration voltage U_1 and fixed braking voltage U_3 , the current I_A is measured as a function of the accelerating voltage U_2 . Figure 4 shows a typical Franck-Hertz curve. One notices that the current sharply decreases for a voltage $U_2 \sim 5 V$ and then increases up to $U_2 \sim 2 \times 5 V$, where a new, similar decrease takes place again. At values corresponding to $U_2 \sim n \times 5 V$ one observes peaks (maxima) in the output curve signal, thus producing a recurring pattern of approximately periodical maxima and minima with increasing amplitudes. The interpretation of these observations is successful with the following assumptions: Having reached an energy of about 5 eV, electrons can transfer their kinetic energy to a discrete excitation state of the Hg atoms. As a result of the inelastic collision, they can not pass the braking voltage. If their energy is twice the required value, i.e. 10 eV, they can collide two times inelastically and analogously for the higher voltages.



Figure 4: Recorded *Franck-Hertz* curve with the setup in the Praktikum. The green and orange arrows indicate the spacing between the minima and maxima. Using the formula $E = e * U \rightarrow 1 eV = e * 1 V$ and the fact that electrons are particles with a charge of 1 (elementary charge $e = 1.602 \ 176 \ 634 \ ^* \ 10^{-19} \ C$), the distance between the extrema can be converted from Volts (V) to electron Volts (eV).

The energy resolution can be improved when using an indirectly heated cath-

ode and a field-free container. A higher homogeneity of energy of the involved electrons is then achieved. With a more precise measurement setup than the one used in our experiments, detailed structures could be seen in the currentvoltage-plot (i.e. Fig. 5 in Ref. [8]). The additional maxima and minima correspond to further excitation levels of the Hg-atom. The step seen at a voltage of 6.73 eV corresponds, for example, indicates a very intensive line in the Hg-spectrum (184.9 nm, Fig. 2). Not all maxima can be related to observed spectral lines. As discussed in section 1.2, there are selection rules for optical transitions. The observations lead to the assumption that the selection rules for collision excitation are not the same as the ones for optical transitions which explains the fact that some optically forbidden transitions might be observed in the *Franck-Hertz* experiment.

3.2 Technical description of the setup

- U_1 small pre-acceleration voltage, variable between 0 V 5 V, removal of space charges;
- U_2 main acceleration voltage, linearly increasing between 0 V 30 V (output divided by a factor of 10 in order to be recorded/displayed properly by the oscilloscope), a resistance of 100 kΩ confines the mesh grid current in case of glow discharges which can occur if $U_2 ≥ 30$ V;

 U_3 braking voltage, variable between 0 V - 5 V;

 I_A anode current (10e-09 A - 10e-08 A);

Creeping currents and pick-ups of 50 Hz are avoided with:

- grounded screen-electrode S;
- grounded metal foil around the tube;
- DC-current heating of the cathode;
- careful shielding of the cable connected to the DC-amplifier;

ATTENTION: The voltages must not be switched on before the oven has reached its operating temperature (at least 150 $^{\circ}$ C). Otherwise Hg-drops can cause a short-circuit between the tube electrodes.

3.3 Heating of the oven

ATTENTION: Do not heat the oven to temperatures above 220 °C! Higher temperatures lead to the destruction of the tube. Don't make sharp changes in the temperature and in the voltages, you might experiment longer times in waiting for the stabilization of the system temperature and eventually the apparatus can be damaged. Only switch on the voltages after having reached the operating temperature!

3.4 Retrieving the data with LabView

The analog output of the *Franck-Hertz* (FH) electronics box is converted into digital format by an analog to digital converter (ADC) with a standard input between -5 V and +5 V. The ADC is the red box with four inputs connecting the output of the FH electronics box to the PC next to the setup. As default the accelerating voltage U₂ is connected to channel 1 and the output voltage $U_A \sim I_A$ to channel 2. Please note that the actual output from the FH electronics box is divided by ten. This is mainly relevant for the accelerating voltage U₂ in your analysis. The digital data is then retrieved by the PC using a LabView program from which similar graphs as the one shown in Fig. 4 can be recorded. In order to start the program, turn on the computer, log in with your D-PHYS account and open the program "*XYPlotter.exe*" located in C:\scratch\.

- Start the program by pressing on the white right arrow on the top left.
- Select "Ch1" for the "x-axis" and "Ch2" for the "y-axis".
- Select whether you want to divide the voltage of the input channels by 10 by pressing down the knobs on the right box labelled as "Amplification control" for the appropriate channels. Please, note that up

corresponds to no division and *down* to a division by ten. Considering the information provided before that the output voltage of the FH electronics box is already divided by ten, you actually do not want to divide it further and should keep all the knobs *up*.

- Click on "Set digital out" to implement "CH"-changes to the amplification control.
- As a default both x- and y- axes are set to autoscale. If you want to change this, right-click on the y-axis and deselect "AutoScale Y" and right-click on the x-axis and deselect "AutoScale X". A good range for both scales is from -0.5 V to 6 V, but you can easily change it by clicking on the first or last number on the either axis and entering the desired value.
- Start the automatic ramp of the U_2 in the FH electronics box. At this point you should be able to see the development of the FH curve on the graph in LabView.
- Now you are ready to start the measurement by pressing on "Start Measurement".
- Stop the measurement by pressing on "Stop Measurement" once the ramp is finished.
- The program will prompt you to save the measurement. You can save it locally with a meaningful name on this computer or directly on a USB stick. If you save your data locally, please remember to delete it when you are done with the experiment and to save it to an external device.

4 Measurements & analysis

4.1 Measurements in the laboratory

a) First calibration - U_1 : Select a temperature T in the allowed range (between 150°C and 220°C). Set U3 to some value between 0 V and 5 V. Change U_1 until you are able to see something resembling the *Franck-Hertz* curve. Do not worry if the signal is very noisy. The point of this first calibration is to just find a signal which one is then able to optimize with the following calibrations.

b) Second calibration - Temperature T: Set U_1 to the value found in point a) and measure the *Franck-Hertz* curve for 3 different T and 3 different braking voltages U_3 . For each temperature, make a plot with the 3 curves corresponding to each of the chosen braking voltages (e.g. using *MATLAB*). Check how the two parameters influence the quality of the signal (signal-to-noise ratio, equation 5) and estimate for which combination of the two the best signal is reached. To do this, you could use

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}},$$
(5)

with I_{max} and I_{min} being the amplitudes of a given maximum and minimum. If here I_{min} approaches zero, meaning the vertical distance between the xaxis and the minimum in the *Franck-Hertz* curve (Fig. 4) goes towards zero (more signal-to-noise ratio), equation 5 tends to one, which is the theoretical optimal value.

c) Third calibration - U_1 and U_3 : Set T to the optimal temperature found in point b) and measure the Franck-Hertz curve for 3 different U_1 and 3 different U_3 . For each U_1 , make a plot with 3 curves corresponding to each of the chosen breaking voltages. Use again equation 5 to estimate for which combination of the two the best signal is reached.

d) Statistical measurements: Now that the calibration has been performed, you can take a good data-set for the analysis. Take a series of measurements (at least 5), with the calibrated values of T, U_1 and U_3 , as coming from questions a) - c).

4.2 Analysis of your recorded data (at home)

a) Transition energy of Hg: We can estimate the transition energy (in eV or nm) by measuring the difference between the successive minima and also between successive maxima. Do it for your data collected in the last point of subsection 4.1, making statistical use of the multiple repetitions (at least 5). Are these values compatible between each other? Are these values all compatible with the energy of one of the transitions in Hg reported in the level diagram (Fig.2)? Can you identify a trend between the values (do they increase with the order of the maxima, do they decrease, etc.)?

b) Model of Rapior et al.: Rapior et al. (Ref. [7]) present an explanation for the not equidistant spacing of the extrema. Does their explanation apply to your data? They also describe a method to extract the value of the transition energy (equation (4) of their paper) from the data; apply this method to the distances between successive maxima and to the distances between successive minima and extract the value of the transition energy. Is the transition energy extracted from the maxima compatible with that extracted from the minima? Are the extracted values compatible with the energy of one of the transitions in Hg reported in the level diagram? Produce a figure with both the distances between successive maxima and the distances between successive minima as a function of the order of the maximum/minimum and complete with two linear fits according to equation (3) from their paper.

c) Estimation of the *Planck* constant: Use your best estimates of the transition energy and the theoretical value of the wavelength closest to the transition energy you found, in order to get an estimate of the *Planck* constant (use equation 2). Do it for both the transition energy extracted from the minima and the maxima. Are these values of the Planck constant compatible with the literature value? Present your results with respective error margins in a table and compare the deviation to the actual *Planck* constant ($6.62607015 * 10^{-34}$ J s).

5 Report writing and structuring

This section give you an overview of what the report should contain. The reports will be subject to subsequent iterations between the student and the assistant, until a final version is achieved. The report must be structured in a form/layout as follows:

- 1. Title page with student name, date, Experiment title and Abstract;
- 2. General introduction and explanation of the key concepts (Theory);
- 3. Three graphs for the questions b) and c) of section 4.1;
- 4. One combined graph for the question d) of section 4.1;
- 5. One table with the measurements of the distances between the successive maxima and the corresponding transition energies and another analogous table for the distances between the successive minima see question a) of section 4.2;
- 6. One plot of the voltage difference vs order of maximum (minimum), show the points for both the maxima and the minima spacing, report the results as linear fits see question b) of section 4.2;
- 7. A table with the estimated *Planck* constants corresponding to the transition energies found by analyzing the maxima and the minima according to the model of Rapior et al. [7] see question c) of section 4.2;
- 8. Summary of your findings and possible Outlook;
- 9. Applied safety instructions/precautions
- 10. Bibliography;

Scientific reports are often written in $\[mathbb{E}]X$ (Overleaf) which eases the structuring of the document, the typesetting of equations, etc. ... but any format is fine for handing in your report.

References

- Katalog LD DIDACTIC. URL: https://www.ld-didactic.de/phk/ gruppen.asp?PT=VP6.2.4&L=2 (visited on 04/13/2023).
- The Nobel Prize in Physics 1925 NobelPrize.org. URL: https://www.nobelprize.org/prizes/physics/1925/summary/ (visited on 04/13/2023).
- G. F. Hanne. "What really happens in the Franck-Hertz experiment with mercury?" In: American Journal of Physics 56.8 (Aug. 1988), pp. 696– 700. ISSN: 0002-9505, 1943-2909. DOI: 10.1119/1.15503. URL: http:// aapt.scitation.org/doi/10.1119/1.15503 (visited on 03/16/2023).
- [4] Why do electrons not fall into the nucleus? Chemistry LibreTexts. URL: https://chem.libretexts.org/Bookshelves/Physical_and_ Theoretical_Chemistry_Textbook_Maps/Supplemental_Modules_ (Physical_and_Theoretical_Chemistry)/Quantum_Mechanics/09. _The_Hydrogen_Atom/Atomic_Theory/Why_atoms_do_not_Collapse (visited on 04/13/2023).
- [5] Hohn C. Slater. Quantum theory of atomic structure. New York: McGraw-Hill Book Co. Inc., 1960.
- [6] H. Hachisu et al. "Trapping of Neutral Mercury Atoms and Prospects for Optical Lattice Clocks". In: *Physical Review Letters* 100.5 (Feb. 8, 2008), p. 053001. ISSN: 0031-9007, 1079-7114. DOI: 10.1103/PhysRevLett. 100.053001. (Visited on 03/28/2023).
- [7] Gerald Rapior, Klaus Sengstock, and Valery Baev. "New features of the Franck-Hertz experiment". In: American Journal of Physics 74.5 (May 2006), pp. 423-428. ISSN: 0002-9505, 1943-2909. DOI: 10.1119/ 1.2174033. URL: http://aapt.scitation.org/doi/10.1119/1. 2174033 (visited on 03/15/2023).
- [8] F. H. Liu. "Franck-Hertz experiment with higher excitation level measurements". In: American Journal of Physics 55.4 (Apr. 1987), pp. 366–369. ISSN: 0002-9505, 1943-2909. DOI: 10.1119/1.15174. URL: http://aapt.scitation.org/doi/10.1119/1.15174 (visited on 03/16/2023).