Compton Effect

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v2.0

The ^{137}Cs-source must not be removed from the apparatus.

After the completion of the experiment, the detector should be adjusted to the $0°$-position and the output of the collimator has to be closed off by the supplied lead absorbers.

Contents

1 Introduction

The objective of this experiment is to make quantitative measurements of the energy and differential cross section of Compton scattered photons. You will learn how to use a scintillator based detector to measure the energy of photons, how to estimate uncertainties of radioactive processes and how to assess the radiation situation of a practical.

2 Theory

2.1 Energy of the scattered photons

The Compton effect describes the elastic scattering of a photon on a free electron. In this process the photon transfers part of its energy and momentum to the electron. As a result, a shift of the frequency of the photons is observed. The phenomenon cannot be explained in the frame of the classical field theory where the frequency is an attribute of the radiation which should not shift when changing directions. The correct explanation of the effect was given by Compton under the assumption, that the scattering of the radiation flux is identical to the scattering of single photons. The Compton effect therefore validates the corpuscular concept of electromagnetic radiation. For this discovery Arthur H. Compton received the Nobel prize in Physics in 1927.

The effect is most pronounced for the photons with energy in the range $10 - 10^4$ keV. The energy and momentum transfer of the photon to the scattering electron is considerably large. On the other hand, the binding energy of the electron is small, such that the scattering process might be treated in first approximation as an interaction with a free electron.

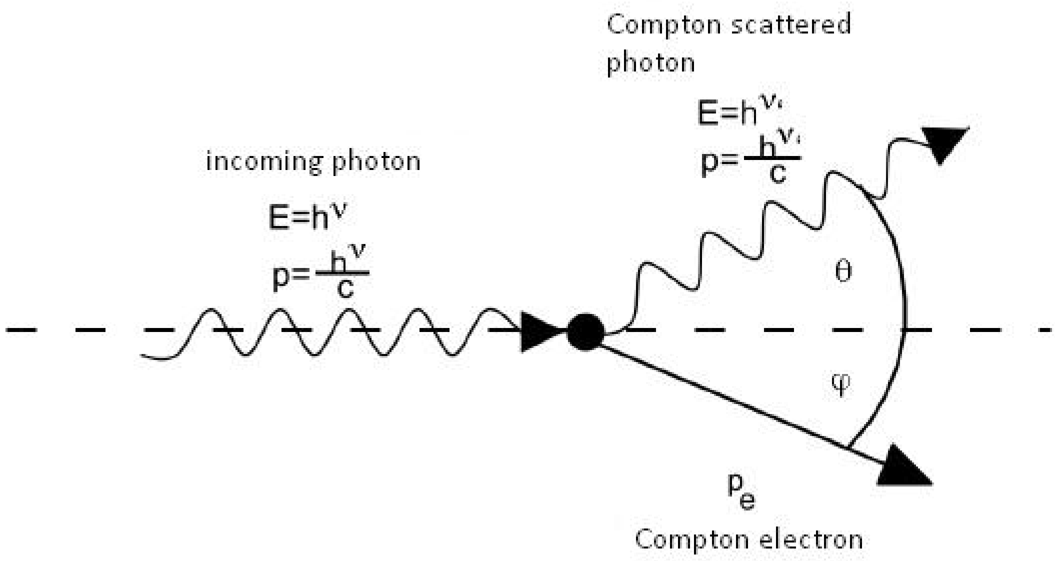

Figure 1: Schematic representation of a photon's scattering on an electron

According to special relativity, for a particle with rest mass m_e and momentum p, it holds that

$$E = \sqrt{\vec{p}^2 m^2 + m_e^2 c^4}. \tag{1}$$

Thus a photon possesses the energy

$$E = h\nu \tag{2}$$

and momentum

$$|\vec{p}| = \frac{h\nu}{c}. \tag{3}$$

In the scattering process the energy and momentum remain conserved. It holds that

$$h\nu + m_e c^2 = h\nu' + \sqrt{\vec{p}_e^{\,2} c^2 + m_e^2 c^4} \tag{4}$$

and

$$\frac{h\vec{\nu}}{c} = \frac{h\vec{\nu}\,'}{c} + \vec{p}_e, \tag{5}$$

where ν' is the frequency of the scattered photon and $\vec{p}_e$ the momentum of the electron after the scattering process. Here we have used that, through a suitable choice of the system of inertia, we can assume that the electron is initially at rest. Applying momentum conservation it follows directly, that the three vectors lie in the scattering plane. Figure 1 shows a schematic representation of these quantities.

By applying the law of cosines to equation (5), one obtains

$$\vec{p}_e^{\,2} = (\frac{h\nu}{c})^2 + (\frac{h\nu'}{c})^2 - 2\frac{h\nu}{c}\frac{h\nu'}{c}\cos\theta. \tag{6}$$

Applying conservation of energy, (equation (4)), one finds

$$\vec{p}_e^{\,2} = (\frac{h\nu}{c})^2 + (\frac{h\nu'}{c})^2 - 2hm_e(\nu - \nu') - \frac{2h^2\nu\nu'}{c^2}. \tag{7}$$

Comparison of equations (6) and (7) results in

$$\frac{\nu - \nu'}{\nu\nu'} = \frac{h}{m_e c^2}(1 - \cos(\theta)) \tag{8}$$

or

$$\frac{c}{\nu} - \frac{c}{\nu'} = \lambda' - \lambda = \Delta\lambda = \frac{h}{m_e c}(1 - \cos(\theta)), \tag{9}$$

where λ and λ' are the wavelengths of the photon before and after the scattering. The constant $\frac{h}{m_e c}$ is the Compton wavelength λ_c.

$$\lambda_c = 2.43 \cdot 10^{-12} m = 0.0243 \mathring{A} \tag{10}$$

The shift $\Delta\lambda$ observed at a given angle θ is independent from the scattering material. For low-energy photons, when $\lambda \gg \lambda_c$, the wavelength of the scattered photon λ' is only slightly greater than that of the incident photon's (in the classical limit). However, when $\lambda \ll \lambda_c$ then $\Delta\lambda$ cannot exceed $2\lambda_c$.

The energy of the scattered photon is

$$E' = \frac{E}{1 + \frac{E}{m_e c^2}(1 - \cos\theta)}. \tag{11}$$

Thus the energy T_e of the electron after the scattering process amounts to

$$T_e = E - E' = E\frac{\frac{E}{m_e c^2}(1 - \cos\theta)}{1 + \frac{E}{m_e c^2}(1 - \cos\theta)}. \tag{12}$$

2.2 Differential cross section of the scattered photons

The differential cross section for the scattering of electromagnetic radiation on electrons is defined by

$$\frac{d\sigma}{d\Omega}_{\mathrm{E}} = \frac{E'}{E}\frac{N'/(t_{\mathrm{measured}}\ \Omega_{\mathrm{detector}})}{j}\frac{1}{N_{\mathrm{e}}}, \tag{13}$$

where N' is the total number of scattered photons, t_{measured} is the time taken for the measurement, $\Omega_{\mathrm{detector}}$ is the solid angle of the detector from the scattering target, j is the incident photon flux on the target (number of photons per unit time and area) and N_{e} is the number of free electrons in the target that can be seen by the detector. This definition of the differential cross section finds the ratio between the scattered and incident energy on the target at a fixed angle θ. The factor of N_{e}^{-1} normalizes this result to the number of electrons available for scattering. Another definition of a differential cross section is the ratio between the number of scattered and incident photons. This differential cross section is given by multiplying equation (13) with the factor $\frac{E'}{E}$:

$$\frac{d\sigma}{d\Omega}_{\mathrm{N}} = \frac{N'/(t_{\mathrm{measured}}\ \Omega_{\mathrm{detector}})}{j}\frac{1}{N_{\mathrm{e}}}. \tag{14}$$

The two definitions are equivalent, but not interchangeable, so you should consider which version is more suitable for your data analysis.

2.2.1 Thomson Formula

We can discuss the differential cross section for photon-electron scattering within the scope of classical theory of electromagnetic radiation. In a simplistic classical treatment, consider the scattering of a linear polarized electromagnetic wave (the incident photon beam) on a free electron. The incident wave can be written as $\vec{E} = \vec{E}_0 \sin\omega t$, where $\vec{E}_0$ is constant in time. The force acting on the electron is therefore given by

$$m_e\frac{d^2\vec{z}}{dt^2} = e\vec{E} = e\vec{E}_0\sin\omega t, \tag{15}$$

where $\vec{z}$ is the position, m_e the mass and e the absolute value of the charge of the electron. The solution of the equation of motion is

$$\vec{z} = -\frac{e}{m_e\omega^2}\vec{E}_0\sin\omega t. \tag{16}$$

The electron moving along this path radiates like an electric dipole. To find the absolute value of the radiated electric and magnetic field at a distance $|\vec{r}| >> \frac{e\vec{E}_0}{m_e\omega^2}$ we can apply electric dipole approximations,

$$|\vec{E}'| = |\vec{H}'| = \frac{k}{|\vec{r}|c^2}|\frac{d^2\vec{p}}{dt^2}|\sin\varphi, \tag{17}$$

where $\vec{p}$ is the dipole moment of the electron, k is the dielectric constant and φ is the angle between $\vec{r}$ and $\vec{p}$. We therefore obtain

$$|\vec{E}'| = |\vec{H}'| = \frac{ke}{|\vec{r}|c^2}\frac{d^2z}{dt^2}\sin\varphi = \frac{ke^2}{rm_ec^2}E_0\sin(\omega(t-\frac{r}{c}))\sin\varphi. \tag{18}$$

You can find the radiated intensity with the Poynting vector $\vec{S} = \vec{E}' \times \vec{H}' = |\vec{E}'|^2$. The intensity is given by the time averaged Poynting vector:

$$I' = \frac{1}{2}|\vec{E}_{max}|^2 = \frac{1}{2|\vec{r}|^2}(\frac{ke^2}{m_ec^2})^2E_0^2\sin^2\varphi. \tag{19}$$

The electron radiates a mean energy of dW to the solid angle $d\Omega$ in the direction φ in a time interval dt. The radiated energy per unit of time and solid angle is given by

$$\frac{dP}{d\Omega} = \frac{d^2W}{dt \cdot d\Omega} = I'r^2 = (\frac{ke^2}{m_e c^2})^2 I_0 sin^2\varphi, \tag{20}$$

where $I_0 = \frac{1}{2}E_0^2$ represents the average intensity of the incident wave. Thus, using the definition given by formula (13), the differential cross section results in

$$\frac{d\sigma}{d\Omega} = (\frac{ke^2}{m_e c^2})^2 sin^2\varphi. \tag{21}$$

We identify the quantity $r_0 = \frac{ke^2}{m_e c^2} = 2.82 \cdot 10^{-15}$ m as the classical electron radius. The Thomson cross section is obtained by averaging over all polarization directions of the incident wave:

$$\frac{d\sigma}{d\Omega} = r_0^2(\frac{1 + cos^2\theta}{2}), \tag{22}$$

where θ is the scattering angle (see Figure 1). We arrive at the total cross section by integrating over all scattering angles.

$$\sigma = \frac{8\pi}{3} r_0^2 \tag{23}$$

2.2.2 Klein-Nishina Formula

The Thomson cross section derived above is independent from the frequency ω, which is in disagreement with the experimental results. That is understandable, because in his evaluations Thomson did not consider relativistic or quantum effects and furthermore neglected the recoil of the electron. The result of an extensive quantum mechanical treatment was given by Klein and Nishina. The derivation is found in Evans [1] and the references therein. The **Klein-Nishina formula** is

$$\frac{d\sigma}{d\Omega} = \frac{r_0^2}{2}(\frac{\nu'}{\nu})^3 \cdot (\frac{\nu}{\nu'} + \frac{\nu'}{\nu} - sin^2\theta) \tag{24}$$

or

$$\frac{d\sigma}{d\Omega} = r_0^2[\frac{1}{1 + \alpha(1 - cos\theta)}]^3(\frac{1 + cos^2\theta}{2})[1 + \frac{\alpha^2(1 - cos\theta)^2}{(1 + cos^2\theta)(1 + \alpha(1 - cos\theta))}], \tag{25}$$

where $\alpha = \frac{h\nu}{m_e c^2}$ represents the ratio of the photon energy and the rest mass of the electron. The reconciliation of equations (11), (24) and (25) with the measurements is a remarkable confirmation of the quantum theory, which shall be reproduced in this experiment.

3 Experiment

3.1 Goals

The goals of this experiment are as follows:

1. Reproduce the Compton effect to find the energy of scattered photons at several scattering angles
2. Reproduce the Compton effect to find the differential cross section of the scattering process at several scattering angles
3. Check for correspondence to the theory and find sources of systematic errors

3.2 Setup

The setup you will use to accomplish these goals is shown in Figure 2. The experiment is conducted with photons of energy 662 keV, which originate from a ^{137}Cs source. The source is mounted on a lead shielding and had the activity of 15 mCi (1Ci = 1 Curie = $3.7 \cdot 10^{10}$ Bg; 1 Bq = 1 Becquerel = 1 decay/s) on 16 April 1975. The half life of ^{137}Cs amounts to 30.17 years. The source is kept in a lead collimator which is sealed with a lead plug seal and lead brick.

Estimate the radioactive activity in the lab and check that it fulfills the ordinance for lab safety (no more than 1 mSv in a year and no more than 1 Sv/h). Refer to appendix A for further details.

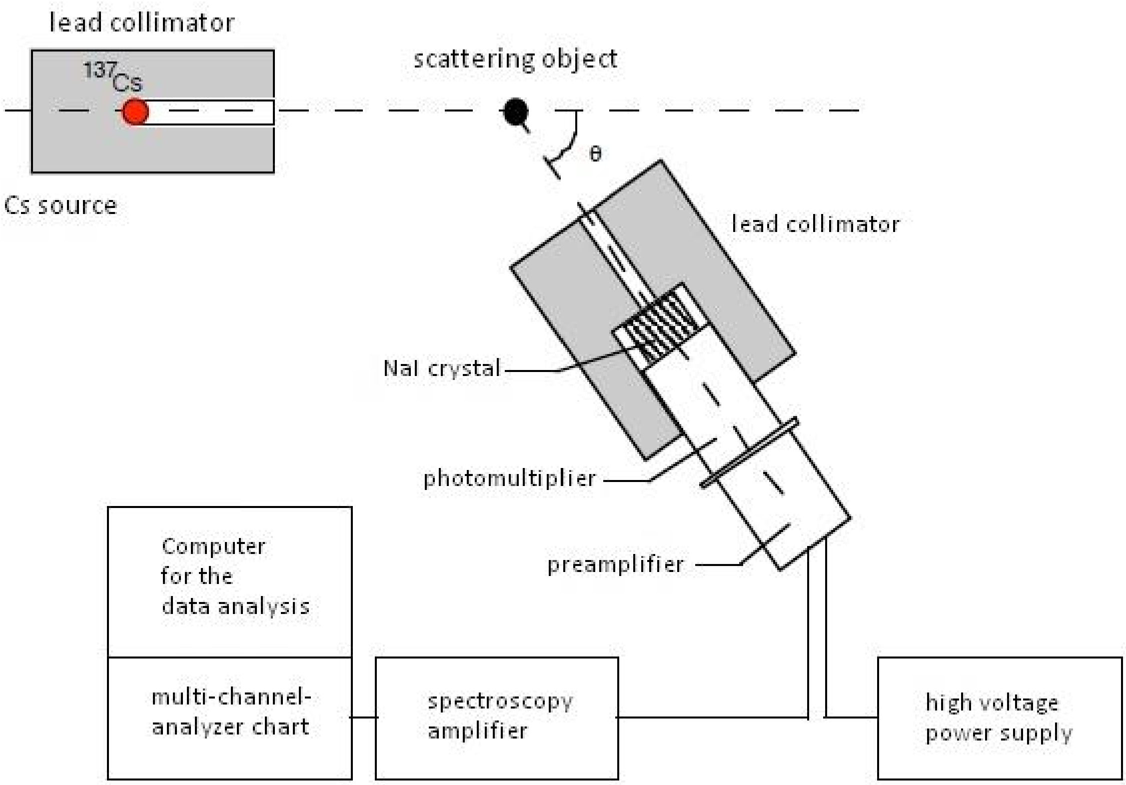

Figure 2: Block diagram of the measurement apparatus

Figures 3 and 4 show, that both the source and detector are mounted in lead collimators, which constrain the solid angle of the detector $\Omega_{detector}$ used in equations (13) and (14) as well as the number of electrons available for scattering, N_e. Therefore, it is important to measure the geometry of the apparatus during your experiment. You may assume that the data given in Figures 3 and 4 is accurate.

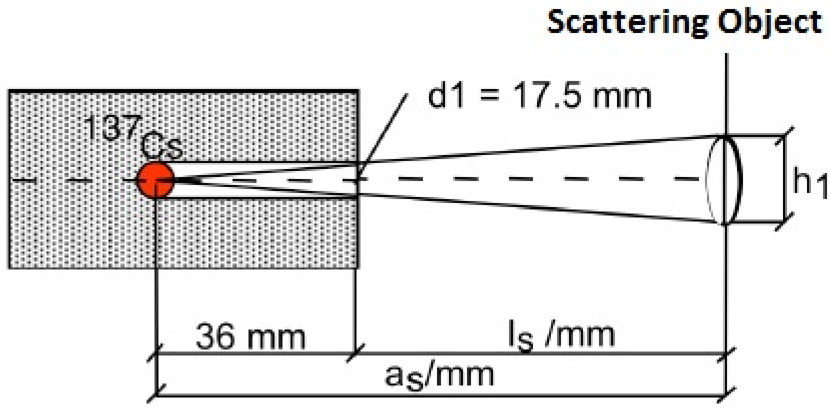

Figure 3: Geometry of the ^{137}Cs source

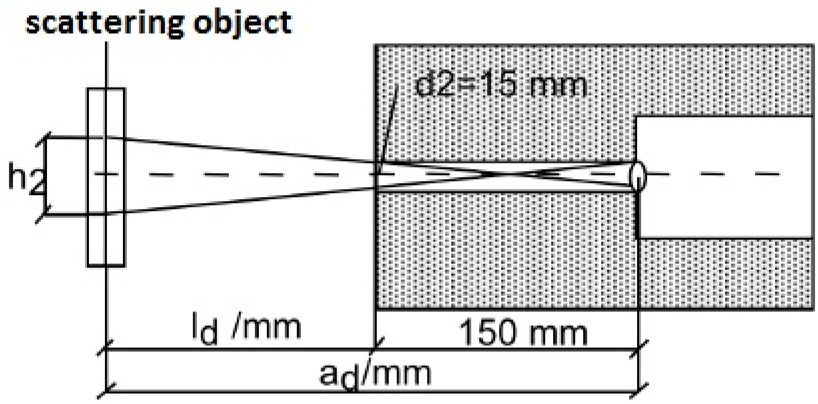

Figure 4: Geometry of the detector construction

Please note that the high voltage supply generally needs around 30 minutes to warm up sufficiently, such that the photomultiplier is stable. Do not adjust the voltage.

3.3 Detector

The photons emitted by the source, that are horizontally scattered at an angle of θ, are focused by the second lead collimator. The detector is made up of a scintillator crystal as well as a photomultiplier tube that is powered by the high voltage source. The photons that reach the NaI(Tl) scintillator are absorbed by the material. The purpose of the scintillator is to produce a large number of low energy photons, which are proportional in quantity to the energy of the initially absorbed photon. Typically, a NaI(Tl) scintillator produces about 4×10^4 photons with average energy 3 eV when a 1 MeV photon is absorbed [2].

The NaI(Tl) scintillator operates on the principle of electron-hole pair production: The incoming photon is absorbed by an electron in the valence band of the NaI crystal, which is then excited into the conduction band (photoelectric effect). The lack of the electron in the valence band due to the excitation is called a hole. This hole can move along the valence band, and the excited electron can move along the conduction band. It can also excite further electrons.

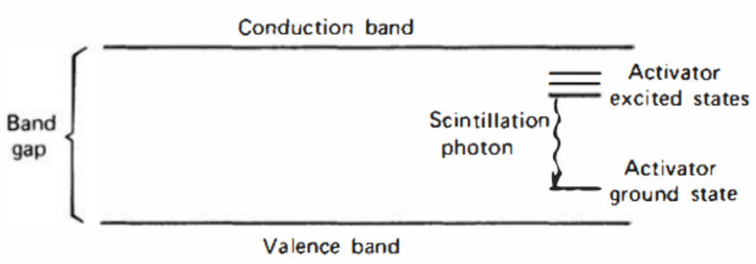

Figure 5: The band model of an NaI(Ti) scintillator. Taken from [2]
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The photons produced by the de-excitation of electrons in a pure NaI crystal are not luminescent, meaning they are not able to pass through the crystal without further interaction, as they have enough energy to excite another NaI valence electron, and will be absorbed in the process. Therefore the NaI crystal in the scintillator is doped with the activator Thallium, which has a thinner band gap between its valence band and conduction bands than NaI. The band model of the doped NaI crystal is shown in Figure 5.

When an electron de-excites from a conduction band of the activator into the valence band of the activator it is possible that the resulting photon is luminescent to the crystal, as it does not have enough energy to excite an electron in the NaI valence band, and it is unlikely to interact with an electron in the activator valence band. The ionization energy of the activator is lower than the ionization energy of the NaI crystal, so once the hole has moved into the valence band of an activator it will not return to the NaI crystal. In terms of Figure 5, you can think of it like minima in the band gap at intervals inversely proportional to the doping concentration.

This makes de-excitation on the activator site possible, as there can be no de-excitation without a hole. Furthermore, the typical lifetime of an electron in an excited state on the activator site is around 30-500 ns [2], so once an electron in the conduction band has moved to the activator site it will de-excite quickly. The activator therefore allows the scintillator to efficiently produce luminescent photons in a quantity proportional to the energy of the incoming photon.

However, not every incident photon is absorbed by a valence electron. A photon could also interact with the lattice through the Compton effect (the attenuation coefficients for the scintillator can be found in Appendix B in Figure 11). During the scattering process the photon transfers some energy to the electron. If the photon leaves the lattice after scattering, only the energy transferred to the electron due to the Compton effect will be measured. As a consequence, the final energy spectrum produced by the computer has two defining features: Firstly, it shows a peak produced by incident photons that were absorbed through electron-hole pair production. This is called the photopeak, and indicates the energy of the photons incident on the detector. Make and justify an assumption on the nature of this distribution. Secondly, the energy spectrum shows a large amount of noise for a wide range of energies smaller than the photopeak caused by a photon leaving the lattice undetected after Compton scattering within the scintillator. This region is called the Compton continuum.

In an ideal detector with infinite energy resolution, the photopeak would resemble a Dirac delta distribution at the scattered photon energy. The Compton scattered photons which leave the lattice would appear in a Compton continuum with a maximum energy determined with equation 11 at a scattering angle of $\theta = \pi$. The ideal energy spectrum can be seen in Figure 6. However, the detector in a real experiment is not ideal. A typical energy spectrum you will measure is shown in Figure 7.

A third possible reaction of photons in matter would be pair production of an electron and positron. Why is this not relevant for this experiment? Consider Figure 11 in Appendix B.

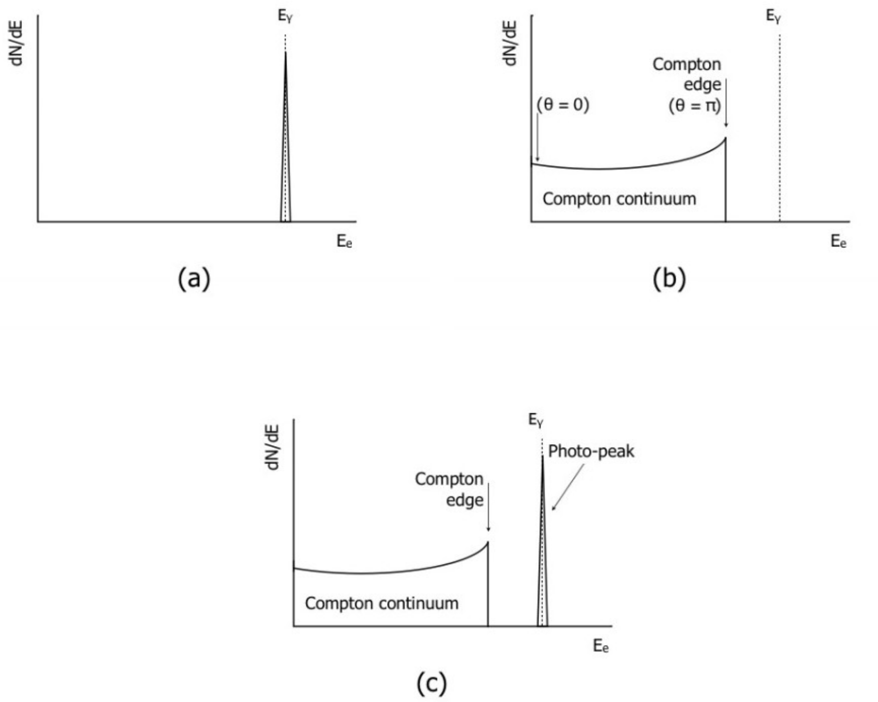

Figure 6: Sketches of the detected energy spectra for photoelectric absorption (a), Compton scattering (b), and both interactions (c), in the approximation of small detectors and infinite energy resolution.

Since the attenuation coefficients (see Appendix B for further explanation on these) for the Compton and photoelectric effect in the target depend on energy, the proportion of events in the photopeak to the total number of events is also dependent on the energy. This dependency is shown in Figure 8.

There is also a possibility that a photon does not interact with the scintillator at all. The overall probability for detection is dependent on the energy of the incident photon, as well as the dimensions of the detector. Figure 9 shows this relationship for several detector dimensions.

Some of the luminescent photons produced by the scintillator reach the photocathode at the front of the photomultiplier (PMT), and are then absorbed by the photocathode through the photoelectric effect, which produces free electrons. These electrons are multiplied and accelerated by the PMT using the high voltage power supply. The quantity of the electrons produced remains proportional to the energy of the incoming photon. The resulting electric signal is large enough to be detected by the amplifier, which clears out noise and strengthens the signal. It then outputs a distinct pulse in unipolar or bipolar form. The unipolar signal is also fed into to the computer, which calculates the dimensions of the pulse. These are used to determine the initial energy of the scattered photon.

3.4 Amplifier

The electrical pulse produced by the PMT is strong enough to be interpreted and strengthened by the preamplifier, which then passes the signal to the spectroscopy amplifier, which in turn converts this pulse into a clear unipolar or bipolar signal. It does this by reducing signal to noise ratio, recognizing pileup, minimizing the fall time and leveling the base signal to zero after a pulse.

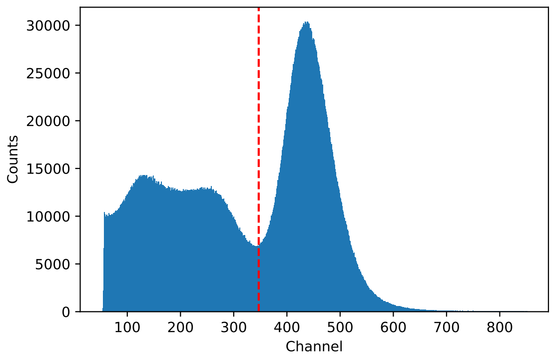

Figure 7: Typical energy spectrum measured at a scattering angle of 0° degrees. The dashed line indicates the separation between the photopeak (right) and the Compton distribution (left).

The amplitude of the resulting pulse indicates the energy of the incoming photon. Pileup occurs when two pulses are measured nearly simultaneously and therefore overlap. The amplitudes of the pulses measured by the multi-channel analyser are therefore larger than they would be for the single pulses, or the second pulse may not be detected at all.

The signal before and after the processing done by the spectroscopy amplifier should be measured with an oscilloscope. You can find explanations on the nature of the signals in Knoll [2]. The rise and fall times of each signal should be determined. If photons were detected in regular time intervals, what would be the maximum possible frequency of detection without pileup for each signal? Given that nuclear decay is Poisson distributed in time and using the activity of ^{137}Cs source (see chapter 3.2), find the probability that the unprocessed/processed pulse contains pileup. You should also determine the reduction in signal-to-noise ratio due to the spectroscopy amplifier.

3.5 Calibrations

The original data produced by the computer is in the form of a histogram: It shows the data produced by the multi-channel analyzer, which bins photons of similar energy into a channel, and counts the number of such photons detected. The energy range of every channel is unknown, and due to the various processing steps within the detector it is impossible to determine a theoretical proportionality constant of the energy. Instead, you will use a source with a known energy spectrum to assign energy values to some channels.

The nature of the proportionality between the channel number and the energy of the photons is to be determined. The energy spectrum of an ^{152}Eu source, which has several peaks, should be used. In order to correctly identify these peaks, measure the energy spectrum of the the ^{137}Cs source first. Identify the 662 keV peak and assigned a channel to this energy. By assuming that the calibration is roughly linear with a negligible offset, attribute an energy estimation to every channel. Then measure the energy spectrum of the ^{152}Eu to identify some of the expected peaks (122 keV, 245 keV, 344 keV, 779 keV, 964 keV, 1112 keV and 1408 keV). In addition, narrowing the calibration to your region of interest can help. Make sure to recalibrate several times during your experiment to account for drift.

3.6 Estimation of efficiencies

The efficiency of the detector is given by the efficiency of the NaI scintillator, $\epsilon_{\mathrm{NaI(Tl)}}$, the efficiency of the PMT, ϵ_{PMT}, and the ratio of counts that are in the photopeak, $\epsilon_{\mathrm{Photopeak}}$. The total number

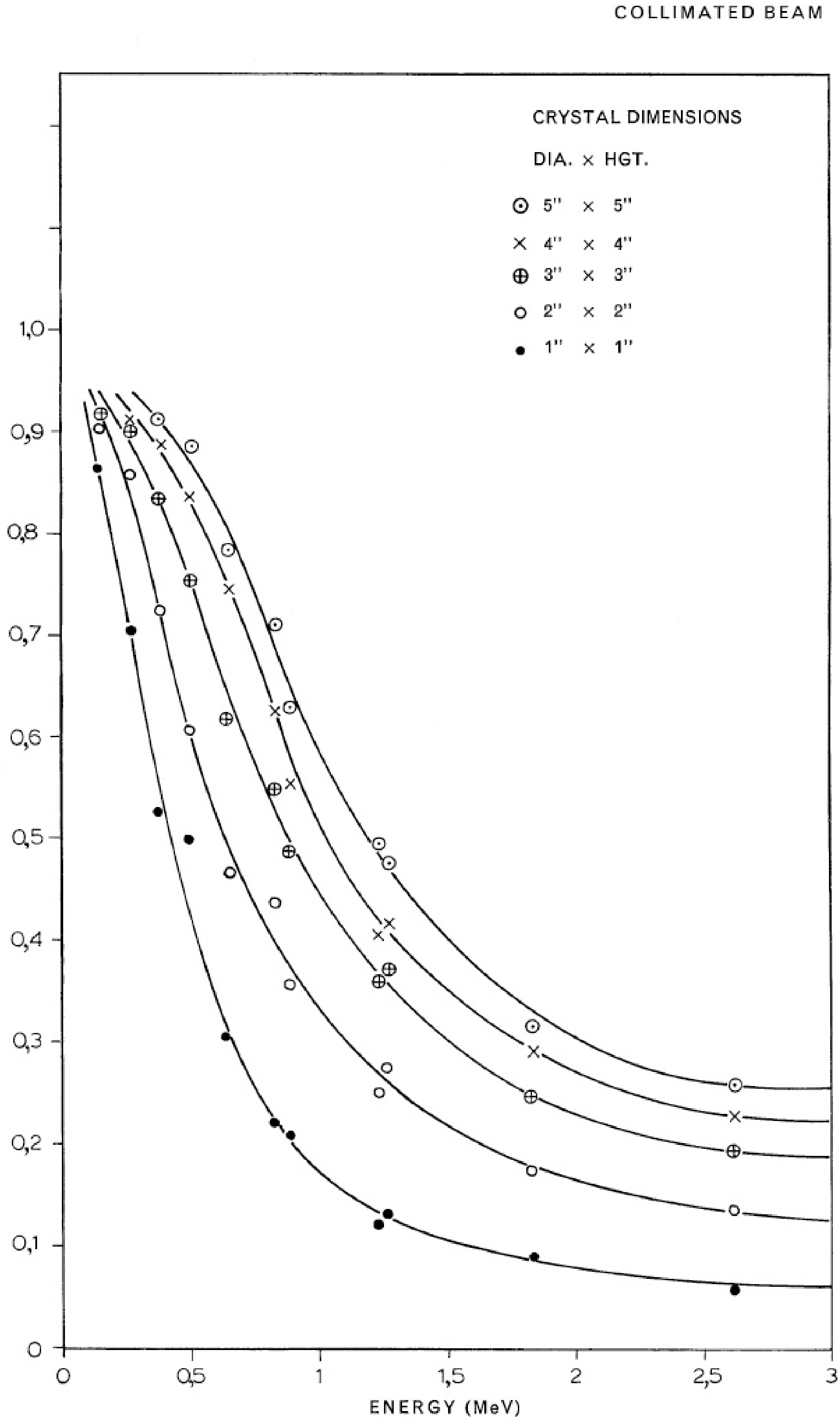

Figure 8: Ratio of the events in the photopeak to the total number of events.

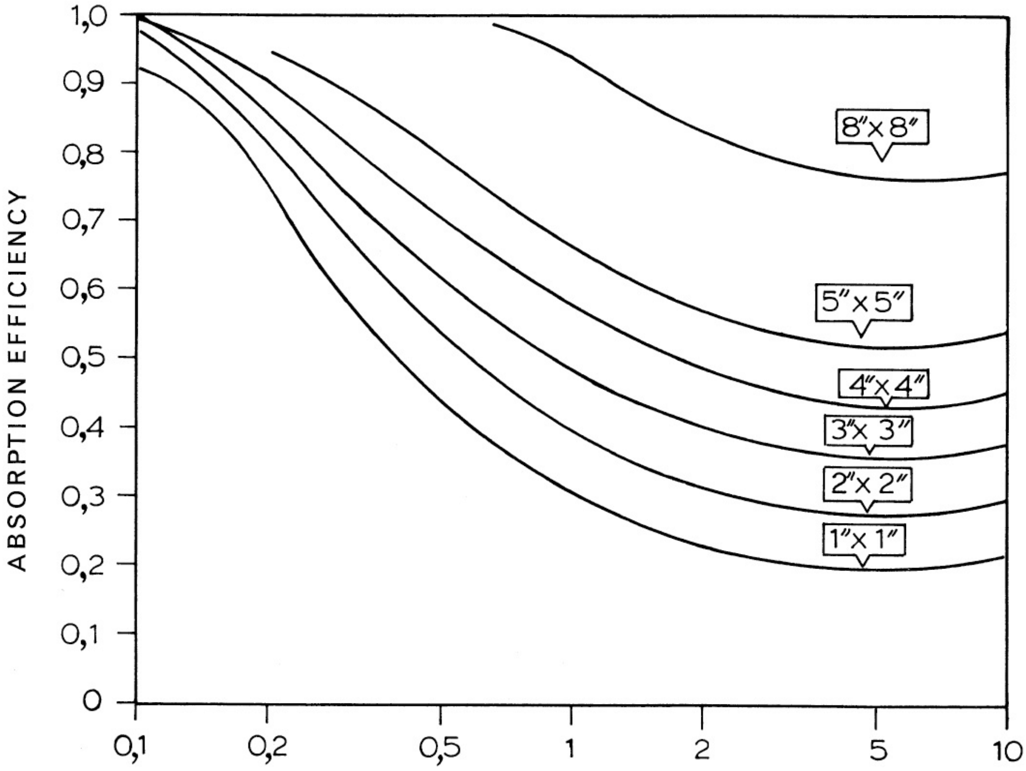

Figure 9: Total detection probability for different NaI scintillators. The x-axis shows energy in MeV.

of measured photons N_{measured} as a function of incoming photons N_{in} is therefore given by

$$N_{\text{measured}} = \epsilon_{\text{NaI(Tl)}}(E_{in}) \cdot \epsilon_{\text{Photopeak}}(E_{in}) \cdot \epsilon_{\text{PMT}} \cdot N_{\text{in}}. \tag{26}$$

As the dimensions of the NaI scintillator is unknown, it is not possible to read these values of Figures 8 and 9 directly. The efficiency of the PMT is also unknown. Therefore, use the fact that you know the exact energy of the photons from the ^{137}Cs source, and measure the energy spectrum of this source directly. Do this several times, while varying the distance between the source and the detector. Compare the ratios of measured events in the photopeak to total events to the values suggested in Figure 8. What are the dimensions of your NaI(Tl) scintillator crystal? You can also determine the efficiency of the photomultiplier using this data set.

Additionally, compare the theoretical and experimental results for the relationship between the rate of incident photons and the distance of the source to the detector. If there is a significant amount of pileup, the detector will not be able to detect more photons when moved closer to the detector. Is there evidence for pileup?

3.7 Energy measurements

To measure the energy of the scattered photons, use a cylindrical lead target. Use Figure 10 in Appendix B to determine the probability that a photon will interact with the target through Compton effect. For an explanation on how to use attenuation coefficients, see Appendix B. What does the thickness of the target therefore imply about the time you need to measure the scattering for?

Once a photon had been Compton scattered once, there is a chance that it will scatter again. The energy of the double scattered photons is likely to be significantly lower than that of a photons scattered only once. Is it important to consider double scattering in this part of the experiment?

Measure the energy spectrum of the scattered photons for several scattering angles between 30°and 120°. Also measure the energy spectrum of the background radiation. Identify the photopeak and find the energy of the scattered photons at each scattering angle.

3.8 Differential cross section measurements

To measure the differential cross section, use a thin cuboid target with a layer of PMMA (Poly(methyl methacrylate)). What is the magnitude of the probability of double scattering in this target? Assume the first scattering occurs at the most likely angle only (0°) and that the photon is equally likely to scatter at any point in the target. Use reasonable approximations for any further information you need. Does this probability justify the use of the thin target? Why is it important to avoid double scattering in this part of the experiment?

The drawback of using a thin target is that the probability of a photon experiencing the Compton effect when passing through the target is lower, as the number of electrons available for scattering (N_e) is lower. Therefore, before beginning your measurements, estimate the amount of time you should measure each scattering angle for in order to get results with a reasonable uncertainty. To find the number of scattered photons relevant for the differential scattering cross section, consider the efficiency of the detector. You may use the questions in the preparation as a guideline. Also make sure to perform background measurements for each scattering angle.

4 Preparation

To help you plan your experiment efficiently, here are some preparatory questions. To solve these questions, an overview of counting statistics, radioactive decay and radiation detection in scintillators will be useful. For an explanation of these, you can refer to Knoll chapters 3, 8, 9, 10, 16 and 17 [2]. The geometry of the setup is as shown in Figures 3 and 4.

1. Using equation (11) in the manual, compute the ratios of $\frac{E^{\prime}}{E}$ for several scattering angles (for example: 20, 40, 60, 70, 80, 100, 120).
2. Using that ratio and equation (24) in the manual compute the differential cross section of a single photon for the same set of angles.
3. The source had the activity of 15 mCi (1Ci = 1 Curie = $3.7 \cdot 10^{10}$ Bg; 1 Bq = 1 Becquerel = 1 decay/s) on 16 April 1975. The half life of ^{137}Cs amounts to 30.17 years. Compute the current activity of the source.
4. Compute the rate of photons from the radioactive source per square cm at the distance of 24 cm from it. How does it increase if you reduce the distance to 20 cm?
5. Using the schematic drawing of the collimator of the detector in Figure 4 and assuming a_d = 30 cm compute the solid angle of the detector as seen from the centre of the target.
6. Using the differential cross section you obtained in step 2, the number of photons per square cm per second at the target you computed in step 4, and equation 24, compute the number of photons scattered per second into unit solid angle.
7. Again, using Figure 4, compute h_2 (the height of the target seen by the detector)
8. Using h_2, assuming a target thickness of 1 mm and a target width of 1 cm and assuming that target material is lead, compute the number of electrons in this volume of the target.
9. The target contains an extra 2 mm thick sheet of acrylic glass. Repeat the calculation for the target Pb + PMMA.

10. Using the number of scattered photons you computed in step 6, the solid angle in step 5, and the number of electrons in step 8, compute the rate of the photons incident on the detector.
11. Using the rate you computed in step 9 and assuming a detector efficiency 20% compute the rate of events you will measure in total
12. Assuming that only 10% of the photons are detected via photoelectric interaction, compute the number events in the photopeak.
13. Given the rate of events in the photopeak, compute how long you would need to collect events for to have a 10% uncertainty. You may assume that the photons are normally distributed within the photopeak. Hint: The standard error of a normal distribution is given by $\sigma_x = \frac{\sigma}{\sqrt{n}}$, where σ is the standard deviation and n is the total number of events.
14. Assuming that the background rate in the peak region is 0.005 counts per second and using error propagation for formula $N_{\text{signal}} = N_{\text{total}}$ - $N_{\text{background}}$ compute how long you need to collect data to have 10% uncertainty. You may assume that you have measured the background rate for the same amount of time as the total rate. What happens if the background rate increases by a factor of 10 or 100?

References

[1] Robley Evans. *The atomic nucleus.* McGraw Hill Book Company, Inc., 1956.

[2] Glenn F. Knoll. *Radiation Detection and Measurement.* John Wiley & Sons, Inc., Danvers, MA, fourth edition, 2010.

A Radiation situation in the lab

The activity of the radioactive probe is represented in the unit Becquerel (Bq). The number of decays per unit of time defines it: 1 Bq = 1 decay s^{-1}. Becquerel replaces the early unit Curie(Ci). 1 Ci = $3.7 \cdot 10^{10}$Bq.
The impact of the radiation field on air or other matter, especially on body tissue, is determined by the dosage of the radiation exposure. These are: energy, equivalent and effective dose.
The energy dose (D) is defined by the deposited energy into one unit of mass by ionizing radiation. The particular unit is Gray (Gy); 1 Gy = 1 J/kg. Prior the use of another unit, rad, was common: 1 rad = 10^{-2} Gy. As the radiation field can consist of various components R, where different weighting factors w_R are associated to their biological radiation impact on a tissue or an organ T. We introduce the equivalent dose H_T, $H_T = \sum_R w_R \cdot D_{T,R}$. The unit of the equivalent dose is Sievert(Sv); 1 Sv = 1 J/kg.

Radiation type	**energy interval**	**weighting factor w_R**
photons	all energies	1
electrons, muons	all energies	1
neutrons	less than 10 keV	5
	10 keV - 100 keV	10
	100 keV - 2 MeV	20
	2 MeV - 20 MeV	10
	more than 20 MeV	5
photons, without repulsed photons; α-particles; heavy nuclides	more than 2 MeV	5
decay fragments	all energies	20

To describe the different damaging effects of a radiation field on the human body, its tissue and organs by ionizing radiation, an effective dose E is needed. We assign a weighting factor w_T to every organ and tissue. Compute the effective dose as folows: E = $\sum_T w_T \cdot H_T = \sum_T w_T \cdot \sum_R w_R \cdot D_{T,R}$. The unit of the effective dose is also Sievert(Sv).

$D_{T,R}$ = dose R absorbed by tissue T
w_T = Weighting factor for tissue (portion of the total risk for tissue/organ T)
w_R = Weighting factor of radiation R
H_T = equivalent dose of the tissue/organ T

Tissues / organs	**weighting factor w_T**
gonads	0.20
bone marrow (red)	0.12
colon	0.12
lungs	0.12
stomach	0.12
bladder	0.05
breast	0.05
liver	0.05
esophagus	0.05
thyroid	0.05
skin	0.01
bones	0.01
rest	0.05

For people, who are exposed to radiation higher than the natural background, not from controlled sources, which is not work-related or due to educational work, there are safety limits for not professionally radiation exposed people. In this case an effective dose of 1 mSv per year must not be exceeded. Find out, whether that is the case in this experiment, assuming that the measurement apparatus is not misused. A dose measuring device is provided for the experiment.

B Linear attenuation coefficients

When going trough matter, the intensity of electromagnetic radiation attenuates. Let $N(x)$ be the number of γ-quanta, which hit on a layer of thickness dx at level (depth) x. The number of interactions in the concerning layer equals to the decrease of the primary quanta.

$$dN(x) = -N(x) \cdot \sigma \cdot n^{\circ} dx \tag{27}$$

The total cross section per atom σ can be represented as a sum of all involving processes:

$$\sigma = \sigma_{photon} + \sigma_{Compton} + \sigma_{Pair} \quad [/cm^2] \tag{28}$$

By multiplication with the atom density n°/atom/cm^3 we obtain the linear attenuation coefficients

$$\mu = \sigma \cdot n^{\circ} \quad [cm^{-1}] \tag{29}$$

Integration of equation (27) results in the number of primary quanta, that penetrate the layer of thickness d without any interaction

$$N = N_0 \cdot e^{-\mu \cdot d} \tag{30}$$

Thereby μ or σ depends naturally on the quantum energy. A fundamental quantity is the mass attenuation coefficient M, its reciprocal X_0 is denoted as radiation length.

$$M = \frac{\mu}{\rho} = \frac{L}{A}\sigma \quad [cm^2 g^{-1}] \tag{31}$$

where L = the Avogadro number and A = atomic mass.

$$N = N_0 \cdot e^{-M \cdot (\rho \cdot d)} \tag{32}$$

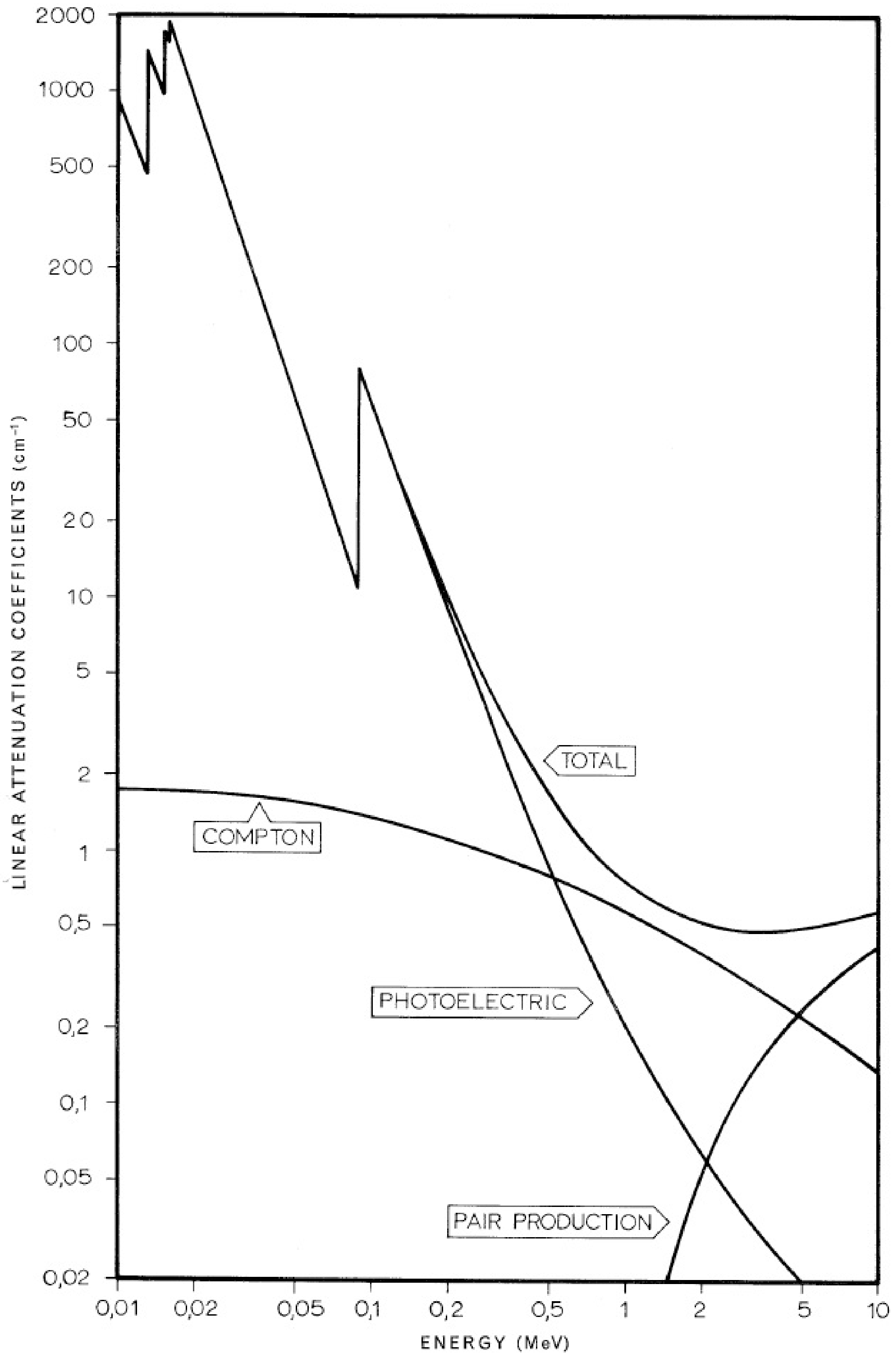

Figure 10: Linear attenuation coefficients of Lead.

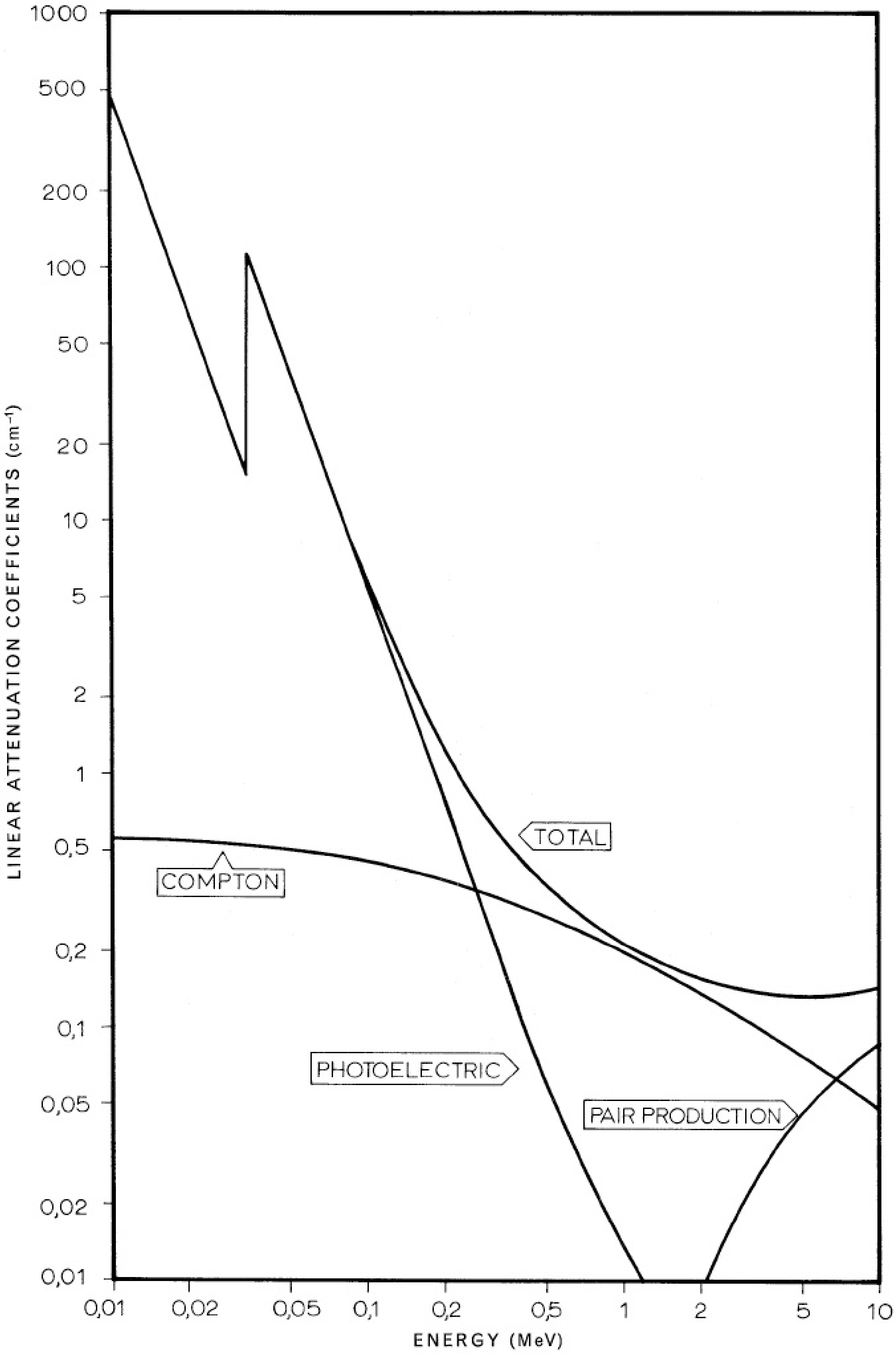

Figure 11: Linear attenuation coefficients of NaI.